SYSTEMS
IN
DEFENCE

If you think that traditional defensive methods are good, you are sadly mistaken !

This book contains new, significantly better methods,
and demonstrates why they are better.

Use Combine Leads.
Use the Mixed Signal.
Try SEQUEL.

Don't miss this chance !
10 years ago Weak Opening Systems were also little known.

by Łukasz Sławiński

FOREWORDS

FROM THE FOREWORD TO THE FIRST EDITION

The area of defensive signals is still a relatively unexplored one in bridge. In choosing one method rather than another we are usually influenced by the current fashion, years of habit and other similarly irrational reasons. This book is an attempt to create a theory of defensive signals, to examine known methods and to construct better ones. In order to do this the author has used a method of statistical analysis which first appeared in the article "Distributional Leads" (Brydz, 9/1974).

FOREWORD TO THE SECOND EDITION

The first edition surprised many readers, who expected to find the answers to such problems as "What do you lead against 3NT after such-and-such a sequence", "When to try for a trump promotion", etc. Some readers thought that, as I had decided to publish my own discoveries in the field of defence, I should have limited myself to a short pamphlet rather than a comprehensive book. So I want to make it clear that the aim of this book is not only to answer the question "How should I defend ?" but also, and more importantly, to answer the question "Why should I defend this way ?".

Any worthwhile discovery can be made by accident: "We've been playing this for a while now; let's try that" (and next month we'll play so-and-so). One of the methods you try may well be the best available; but what of it if you don't realise it and abandon it after the first disaster in favour of tried and trusted methods. So it is not enough to make a discovery; you also have to prove that it is a good one. The way to do this is not to produce examples where your method works – there will also be examples where it does not work – but to investigate it statistically. Results in practice are certainly not the right criterion, and on this point I am in agreement with Zachary Lichter:- "The desire to find a unique solution to a problem is not a function of stupidity, the strength of which depends on the fact that it can accept any theory, even a demonstrably bad one, provided good practical results can be obtained using it."

In the second edition, certain areas were examined in more detail, and some new ideas were introduced (evaluation of signals, permutations of small cards, SEQUEL, problem 5 – 6, expressing the efficiency of a system as a percentage). The name "Combination Leads" (or "Mixed Leads") was changed to COMBINE (the adjective mixed being reserved solely for the Mixed Signal and MM). In the Combine System itself the leads from Q 10 9 and Hxxxxx were changed.

FOREWORD TO THE THIRD EDITION

The third edition differs only slightly from the second (a few minor corrections). In the Combine System the lead from six small cards has been changed from ⓧxxxxx to xⓧxxxx. An addition to the chapter dealing with permutations of small cards is Marek Dryanski's idea about minimising the rank of the first small card played. Also, the relative efficiency of the signals L Q M has been corrected (see 'Evaluation of signals') as well as a few small errors in the test tables. This has the effect of insignificantly changing the percentage efficiency of some small-card systems, but has little bearing on the final conclusions.

In the 4th edition the author hopes to achieve totally error-free results with the aid of a computer.

CONTENTS

PART ONE

HONOUR SYSTEMS

or signalling sequences

WHAT AN HONOUR SYSTEM IS

The lead of an unsupported honour occurs only rarey. The reason is simple: it is much safer to lead away from an honour, and just as attacking. So an unsupported honour is normally led only in these specific cases:-

1) From a doubleton (so as not to block the suit)
2) When you want to stay on lead and decide on the best defence after seeing dummy
3) When partner has bid the suit, and you want him to know about your honour.

The situation changes completely, however, when the honour you possess is part of a sequence (KQJ, QJ9, Q109, AK etc.). Then the lead of an honour will, on balance, show a profit, as the risk of losing a trick is considerably lessened, and the chance of establishing a trick correspondingly greater. Obviously, you will be happiest leading from a full three-card sequence (AKQ, KQJ, QJ10, J109); a good lead with no risk. But sequences like that occur comparatively rarely, so often (but less willingly) you decide to make a riskier lead from an incomplete sequence (KQ10, QJ9, Q109 etc.), or even from a two-card one (AK, KQ, QJ etc.). So it is clear that the following popularly accepted statement is true:-

The lead of an honour implies possession of a sequence.

However, there are only six honours (AKQJ109), whereas there are considerably more sequences. This gives rise to the problem:-

Which honour should be led from a given sequence ?

If we chose an honour to lead at random, partner would know we probably had a sequence, but he would have no idea which one. For example, seeing the jack led, he could assume it was from KQJ, or AJ10, or J109, or QJ9.......the most likely result being that he would fail to find the right defence. To avoid this any partnership must decide:-

Which honour do we lead from any given sequence ?

The answer to that is what an honour system consists of.

PRELIMINARY ANALYSIS

Types of sequences

A sequence consists of two touching honours (AK, KQ, QJ, J10, 109) with or without a third honour. We can distinguish between the following types of sequences:-

Full sequence	AKQ KQJ QJ10
Broken sequence	AKJ KQ10 QJ9
Short sequence	AK KQ QJ J10 109
Interior sequence	AQJ AJ10 KJ10
	A109 K109 Q109

Obviously, there may be a number of small cards attached to any of these sequences, but this does not concern us at present. Some sequences, such as J109 or KQ9, have not been singled out, as in practice they can be treated in the same way as similar sequences, so J109 is equivalent to J10 for practical purposes, KQ9 to KQ, etc.

The relationship between sequences

The first three types of sequences are clearly related to each other : a full sequence changes to a broken one by replacing the lowest honour with one a step lower; to a short sequence by removing the lowest honour.

Schematically:-
AKQ KQJ QJ10
↓ ↓ ↓
AKJ KQ10 QJ9
↓ ↓ ↓
AK KQ QJ

Interior sequences are not so clearly related to the others and form a separate group.

Simplified Honour Systems

A short sequence is a member of a class consisting of itself and two related sequences – broken and full (e.g. sequence KQ represents the class KQ KQ10 KQJ). It is often enough to know which class, rather than which specific sequence, partner has led from. For example, to know that partner has KQ (and perhaps the J or 10) is acceptable as the presence of the additional honour is in many cases irrelevant. Thus we can define a simplified honour system as one which distinguishes only between two types of sequences:-

Short :- AK KQ QJ J10 109
Interior :- AQJ AJ10 KJ10
A109 K109 Q109

Full and broken sequences are equivalent to short sequences, e.g. KQ10 or KQJ is treated in the same way as KQ.

Complete Honour Systems

This name is reserved for honour systems which distinguish between all types of sequences.

General assumptions

Let us make the following general assumptions:-

1) The aim of a system in defence is to transmit maximum information to partner.
You might say that a system which gives too much information makes things easy for declarer; this is undoubtedly true, but if you have led the wrong suit it will generally make no difference, whereas if you have led the right one it is vital that partner is aware of the true situation.

2) A system in defence should be the same against both suit and no-trump contracts.
Obviously, you could use umpteen different systems in defence depending on the denomination of the contract, the level of the contract, the course of the auction etc.; but who could remember it all ? It must be better to use one system all the time. If it is a good one, all supercomplications are pointless; they may increase your effectiveness by 1%, but will also increase the amount of effort needed by 100%.

3) You are allowed to lead from any honour sequence.
It is hard to take statements like "Never lead away from a king" seriously; if, in your judgement, it is right to lead a given suit, you should lead it, even underlead an ace.

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The above assumptions (especially not distinguishing between a suit contract and a no-trumpcontract) may seem too radical for many readers; but please finish reading the book before you make a judgement.

TRADITIONAL HONOUR SYSTEMS

By this I mean known methods of signalling sequences:- the Culbertson method, the "normal method and the Rusinow method.

The Culbertson method

The lead of an honour denotes the following sequences:-

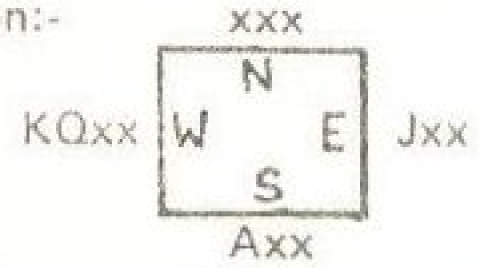
A = A
K = AK or KQ
Q = QJ or AQJ
J = J10 or AJ10 or KJ10
10 = 109 or A109 or K109 or Q109

As you can see, the lead of the ace is the only one which does not show a sequence. Why ? It has already been said that an unsupported honour is sometimes led in order to stay on lead and, after seeing dummy, choose the most appropriate continuation (often just cashing top tricks). Clearly, the best chance of doing this is to lead the highest honour – the ace. As the sequence AK is significantly less common than an unsupported ace, the Culbertson method has its advantages: partner will only play an encouraging card if he has the king, so if he does not encourage you know you have to switch. Also it is easier to defend speculative 3NT contracts (when you may have five top tricks to cash) and high-level contracts.

Ambiguity in the Culbertson method

The occasional benefit derived from signalling an unsupported ace must, however, be weighed against the annoying ambiguity of K = AK or KQ.

In this situation:-



declarer can put West to a difficult guess by ducking the king. After all, East could not encourage as the lead may have been from AK.

The Normal method

Dislike of the ambiguity in the Culbertson method caused it to be modified as follows:-

A = AK or A
K = KQ
Q = QJ or AQJ
J = J10 or AJ10 or KJ10
10 = 109 or A109 or K109 or Q109

In practice, the ambiguity in the lead of the ace is not so serious, as it is comparatively rare to lead an unsupported ace. In any case, a partnership can agree that Culbertson leads apply in certain specific circumstances.

The Reverse method (Rusinow leads)

The problem of the unsupported ace was solved in this radical manner:-

A = A
K = AK
Q = KQ or AQJ
J = QJ or AJ10 or KJ10
10 = J10 or A109 or K109 or Q109
9 = 109

This method retains the Culbertson lead of the unsupported ace, at the same time eliminating the ambiguity of K = AK or KQ.

Troublesome situations

These occur in all three of the traditional methods described. In the Culbertson method the most troublesome is obviously K = AK or KQ. In the normal method (and also in the Culbertson method) the following situation is very difficult:-

xxx
1) J10x
2) KJ10 Axxx

1) KQx
2) Qxx

On the lead of the jack, the defence can cash three tricks if the situation is as in 2), but if 1) is the case East must switch at once. And in the Reverse method:-

Qxx
1) J10x
2) K109 Axxx

1) Kx
2) Jx

The 10 should be ducked in 1), but in 2) East should rise with the ace. If he does not guess right, he loses a trick or a tempo (assuming a suit contract). In all three methods the following situation is very difficult:-

Jxx
1) Q109
2) A109 Kxxx

1) Axx
2) Qxx

If, on the lead of the ten, East rises with the king, the jack will take a trick in 1), while if he ducks in 2) he may lose a trick (or at best a tempo). Also in the Reverse method J = QJ or AJ10 is unclear; if partner has the king he will have to guess right.

PRINCIPLES OF CONSTRUCTION

What is the crux of the problem ?

By looking at traditional honour systems we see that none of them protects us from difficult situations, where it is hard to read partner's lead, and a wrong guess may mean the loss of a trick. True, we have shown only 4 difficult situations, but more exist. Also, the methods discussed so far are simplified honour systems, i.e. they do not distinguish between broken and full sequences.

Are we then inevitably stuck with bad methods ?

We would be, if we were forced to limit ourselves to traditional methods – top of a sequence or second highest. But nobody can force us to do this ! Let us consider the crux of the whole problem. Honour sequences consist of three (or two) cards from the top six. An honour system is simply an agreement as to which card shows which sequence; so all we have to do is to assign to each of the top 6 cards (AKQJ109) one or more sequences which include it. There are many possibilities, of which we have discussed only three:- Culbertson, normal, and Rusinow. There are, therefore, a lot of possibilities still open to us.

How many honour systems are there ?

From a full sequence (AKQ, KQJ, QJ10) we can lead any card; from the remaining sequences, one of two touching honours. Thus it is easy to calculate that:-

The number of complete systems is 442,368 ($3^3.2^{14}$)
The number of simplified systems is 2,048 ($2^5.2^6$)

So there really is a tremendous number of possibilities.

In either case, there are significantly more sequences (11 in a simplified system, 17 in a complete system) than the 6 cards we have at our disposal to signal them. From this it follows that we cannot avoid ambiguous situations, posing partner the dilemma:- What sequence have we led from ? The important thing, then, is to eliminate the bad alternatives and leave the good ones (or at least the less bad ones). To construct an honour system, it is necessary to examine all possible ambiguities, bearing in mind:-

1) What is the chance of partner knowing which sequence we have led from ?
2) How dangerous will it be if he does not know ?

Some examples of bad alternatives:-

J = QJ or J10
10 = K109 or J10
10 = A109 or 109
9 = Q109 or A109
J = KJ10 or J10
K = AK or KQ
J = QJ or AJ10

And good ones:-

9 = A109 or K109
J = QJ or KJ10
10 = Q109 or KJ10
10 = AJ10 or Q109

ROMAN LEADS AND JOURNALIST LEADS

In the previous chapter we outlined a basis for the construction of honour systems, breaking with the old habits and stereotypes. Similar reasoning was certainly the start of the construction of Blue Club and Journalist leads. Both of these are complete honour systems (or, to be more accurate, semi-complete) and vary according to whether the contract is in a suit or no-trumps.

Here are the leads against no-trumps:-

	BLUE CLUB	JOURNALIST
A =	AKx, AKxx	Asks for unblock or count
K =	AKQ, AKJ, KQJ, KQ10	AK, KQ
Q =	KQ, QJ10, QJ9	QJ, AQJ, (KQ10)
J =	QJ, J10	J10
10 =	AJ10, KJ10, Q109	Interior sequence
9 =	109	109

And against suit contracts:-

Blue Club:-
A = AK
K = AKQ, AKJ, KQJ, KQ10
Others as in the Rusinow method.

Journalist:-
As in the Rusinow method with these exceptions:-

sequence	lead
KJ10	10
K109	9
Q109	9

Both of the above methods have grave drawbacks. Why, for example, does the Journalist system accept the Culbertson ambiguity against no-trumps ? Why does the Blue Club lead of the king give away so much information ? Surely three-card sequences are relatively rare. And why is the lead of the 9 wasted on the uncommon sequence of 109 ? I will not go into a detailed analysis of the above methods, as:-

1) They are not totally complete systems
2) They vary against suits and no-trumps (contrary to one of our initial assumptions)

ALTERNATIVES

As an example, let us analyse some ambiguous situations, where the lead of an honour denotes one of two sequences.

Alternative 10 = Q109 or A109

The chances of resolving this ambiguity on the basis of visible cards (your own and dummy's) are small. Even if you can see the king and the jack you are still in the dark. Does this matter ? It may be that, in spite of not knowing partner's holding, you will know what to do. To convince ourselves of this we have to analyse all possible positions of the king and jack (9 possibilities). Then we will see that in 8 of them, you will have no trouble in making the correct decision (rise with your honour ? duck ? return the suit ?). The only difficult situation is this:-

Jxx
1) Q109
2) A109 Kxxx

1) Axx
2) Qxx

In alternative 1), East should duck the 10; in alternative 2) East should play the king. If he ducks, he may (in a suit contract) lose a trick, and at best a tempo. Because of the possibility of this insoluble ambiguity, we can say that the alternative 10 = Q109 or A109 is definitely a bad one.

Alternative 10 = AJ10 or Q109

The sequences AJ10 and Q109 have only one honour in common -- the 10. Thus the possibility of resolving this ambiguity on the basis of visible cards is very high; as long as any of the AQJ9 are visible, all will be clear. The only time there will be problems is when the king is the only visible honour, or when no honour is visible. Let us analyse these instances:-

I xxx
1) AJ10
2) Q109 xxxx

1) KQx
2) AKJ

This ambiguity will be cleared up immediately as if declarer has holding 2) he will win with the jack, whereas with holding 1) he will play the king or queen.

II Kxx
1) AJ10
2) Q109 xxxx

1) Qxx
2) AJx

In this instance, as in the previous one, declarer's action will tell you what partner's holding is.

III xxx
1) AJ10
2) Q109 Kxxx

1) Qxxx
2) AJx

In either case East should go up with the king, whereupon the card declarer plays will clear the position up.

So we see that the alternative 10 = AJ10 or Q109 is a very good one.

Alternative 10 = J10 or Q109

This will be resolved immediately whenever the Q, J or 9 is visible. The remaining 9 possibilities contain as many as 6 cases when there will be a problem. The most dangerous situation is:-

Kxxx
1) J10x
2) Q109 Axxx

1) Qx
2) Jx

In case 1) East should duck (else he loses a trick); but ducking in case 2) will cost a tempo, and (in a suit contract) maybe even a trick. Because of this and 5 similar situations the above alternative can be classified as very bad.

Comparison of alternatives

Having examined three possible alternatives, we can put them in the following order:-

1. AJ10 or Q109 very good
2. A109 or Q109 bad
3. J10 or Q109 very bad

This can be clarified further by the following table:-

Alternative	Number of honours which resolve the ambiguity	Frequency of difficult situations
AJ10 or Q109	4 (AQJ9)	0/3 = 0%
A109 or Q109	2 (AQ)	1/9 = 11%
J10 or Q109	3 (Q109)	6/9 = 67%

As you can see, this type of analysis is very time-consuming; there are 74 different alternatives in all, and for each of them 9 different cases (on average) need to be examined.

THE COMBINE SYSTEM OF HONOUR LEADS

The author has examined all possible alternatives and the result of his analysis is the Combine system of honour leads. In the author's opinion, this system is the optimum one bearing in mind the general assumptions on page .....

A = AK
K = KQ or AKJ
Q = QJ or KQ10
J = J10 or QJ9 or AQJ
10 = 109 or AJ10 or KJ10
9 = A109 or K109 or Q109

Full sequences (AKQ, KQJ, QJ10) have not been included as they are treated more flexibly:-

Lead the middle honour to emphasise possession of the lowest honour
Lead the highest honour when the lowest honour is likely to be irrelevant

For these reasons it may also sometimes be best to lead the top honour from a broken sequence. With a doubleton sequence (AK, KQ, QJ etc.) against a suit contract it is best to lead the lower honour, implying possession of a third honour, so that partner will be more likely to return the suit, with the possibility of obtaining a ruff. The Combine system of honour leads is obviously not perfect (such a system does not exist), but compared with other systems of honour leads very few difficult situations arise; the most dangerous is the alternative 9 = A109 or Q109. Four years of playing the Combine system (from autumn 1974) have demonstrated its definite superiority over other systems in terms of clarity and information imparted.

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A more detailed description of the Combine system can be found in Part 3.

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.... END OF PART ONE ....

PART TWO

SMALL-CARD SYSTEMS

or

signalling quality and length

WHAT A SMALL–CARD SYSTEM IS

What a small card is

The obvious formal answer is:-

Small cards 98765432
Honours AKQJ10

However, we need a different definition – a more realistic one.

A small card is any card which can be interchanged with any other card in the same suit without affecting the play of that suit.

The above definition rightly differs from what the words "small card" and "honour" bring to mind:-

The rank of a small card has no bearing on its trick-taking capacity
The rank of an honour, in contrast, has a definite bearing on its trick-taking capacity

Some examples to illustrate this:-

In a suit distributed as follows:-

AK96

105 J83

Q742

All cards below the queen are small cards. Any of them can be interchanged with any other, and the result will still be the same: NS will make 4 tricks in the suit. Thus the above diagram can be presented more clearly:-

AKxx

xx xxx

Qxxx

Each small card is denoted by the symbol "x", as its rank is totally irrelevant.

But in this example:-

A105

Q94 J82

K763

the small cards are all those lower than the 8. The 9 and 8 are not small cards, as exchanging either for the 3 (for example) will enable NS to make 4 tricks in the suit if it is led by East or West. So this suit distribution can be denoted as follows:-

A10x

Q9x J8x

Kxxx

The properties of small cards

From the examples given, and our definition of small cards, the following facts emerge:-

1) The boundary between small cards and honours is totally fluid and depends strictly on the distribution in that suit
2) All small cards are equal

What is a working small card ?

The first fact is one which you have no doubt discovered the hard way, having thoughtlessly discarded a small card and found later that it was, in fact, an honour which would have taken a trick. So let us define a "working small card" as one which may turn out to be an honour (even though it is formally a small card) Clearly, the higher the rank of a small card the greater the probability of its being working.

Small cards as sources of information

Taking the second fact, it follows that having decided to play a small card to a trick it is totally irrelevant which one we play, as none of our small cards will take tricks. However, small cards may be equal, but they do possess numbers and can be distinguished. This begs the question – how can we take advantage of the fact that they can be distinguished ? The answer is simple – to transmit information. If we play the lowest small card rather than the highest; if we play our small cards in ascending order rather than in descending order, then partner will be able to draw certain conclusions from the card we play. Obviously, this will only be so if we have agreed beforehand what these signals mean.

What is a small-card system ?

Any kind of information can be transmitted by means of small cards, e.g. hand pattern, number of aces and kings, number of cards in the majors, etc. However, we will not go into these interesting possibilities here; we shall limit ourselves to a basic defensive problem:-

How can one transmit information about the suit played by means of small cards ?

We shall call any such method of transmitting information a "small-card system". We will analyse traditional small-card systems and try to discover the optimum system.

N.B. The General Assumptions on page 6. are still valid.

PRECISION OF INFORMATION

Let us decide how precise information transmitted as to length and quality of a given suit should be.

Length

In practice, on most hands the length of declarer's suits is known approximately, on the basis of the 26 cards visible (dummy and your hand) and the course of the bidding and play. An experienced player will know (and will rarely be wrong) that in any given suit:

declarer has 0 or 1 card
declarer has 1 or 2 cards
declarer has 2 or 3 cards etc.

If he can deduce this about declarer's hand, he can do the same for partner's hand. So it should be sufficient to tell partner whether we have an odd or even number of cards in the suit played. He will be able to work out the exact number by considering the cards he can see, and the bidding and play.

Length problems

The deductions made so far mean that defensive problems can be classed as follows:-

Problem 0 – 1
Problem 1 – 2
Problem 2 – 3 ....etc.

For example, problem 3 – 4 signifies the defensive situation when partner knows you have 3 or 4 cards in a given suit.

Quality

Obviously, length is not the only important factor. Suit quality (number of honours) is equally important. Practical experience has shown that partner cannot determine your suit quality with a sufficient degree of certainty on the basis of available information (dummy, his own hand, the bidding and play), so you must help him by signalling using small cards in a previously agreed way. Because you rarely possess two honours in a suit (the side playing the contract usually has the majority of honours), and because attempting to inform partner as to the rank of your honour would create too many difficulties, you have to be content with telling partner whether your suit is of bad quality (all small cards) or of good quality (any honour).

TYPES OF INFORMATION

Basic possibilities

So far we have singled out two types of information:-

Length = even or odd number of cards
Quality = no honour or one honour

This means that there are four basic possibilities:-

Even number of cards and no honour
Even number of cards and one honour
Odd number of cards and no honour
Odd number of cards and one honour

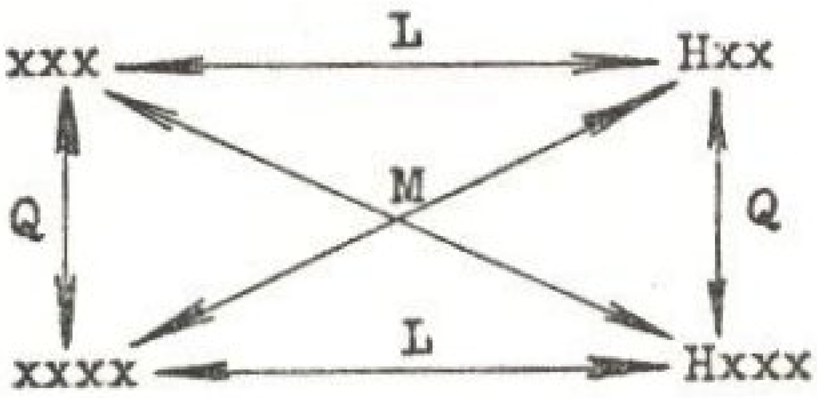
Here is a tabular representation of these possibilities applied to problem 3– 4 (i.e. when partner knows you have 3 or 4 cards in the suit):

	Quality	
Length	bad	good
even	xxxx	Hxxx
odd	xxx	Hxx

where "x" = a small card and "H" = an honour.

Signals

Clearly, it would be best to give partner exact information as to which of the four possible holdings you possess, e.g. "I have xxx", "I have Hxx" etc. Let us forget this ambitious undertaking for the moment and content ourselves with a more modest task: We will give partner ambiguous information of the form "I have xxx or Hxx" or "I have xxx or xxxx" etc. How many ways are there of achieving this ? The following diagram for problem 3 – 4 will show us.



So we see that there are three possible methods, which will be called signals.

Length signal (L) : information about number of cards
odd = xxx or Hxx
even = xxxx or Hxxx
Quality signal (Q) : information about quality
bad = xxx or xxxx
good = Hxx or Hxxx
Mixed signal (M) : mixed information
? = Hxx or xxxx
? = xxx or Hxxx

The mixed signal

The Length signal and Quality signal are traditional signals – known and used for a long time. The Mixed signal, however, came about as a result of purely theoretical speculation, and two questions have to be answered:-

1) What is "mixed" information ?
2) What is its practical use ?

The mixed signal tells partner how many small cards you have:
even = Hxx or xxxx
odd = xxx or Hxxx

As to the second question, some examples will demonstrate:

Problem 3 – 4:
xxx
1. xxx
2. Qxx
3. Qxxx
AJxx
1. KQx
2. Kxx
3. Kx

West leads a small card against a suit contract. East wins the trick with the ace and returns the suit, declarer winning with the king. When East gets in, he is faced with this problem: if West had Qxx there is a trick to take, but if West had xxx or Qxxx he must look elsewhere for tricks. It is evident that neither length nor quality signals would help, as if West is known to hold three cards they may be three small cards, and if West is known to hold the queen it may be queen to four. Only the mixed signal is useful in this case.

Now for an example of problem 2 – 3:

xxxx

1. xx
2. xxx   KQx
3. Jxx

1. AJ10x
2. AJ10
3. A10x

Defending against a suit contract, West led a small card, East played the queen and South won with the ace. Now when East gets in, say with the ace of trumps, he is in trouble: if West had xx – he can get a ruff; if West had Jxx – there are two tricks to take; if West had xxx – even cashing the king could be dangerous, as it sets up dummy's long card. Once again, only the mixed signal is of use, as East knows one of the following:

"I have xx or Jxx" (even number of small cards)
"I have xxx" (odd number of small cards)

It would be best if this information were transmitted by the opening lead.

Now an example of problem 4 – 5:

Qx

1. Jxxxx
2. Jxxx
3. xxxx

A10xx

1. Kx
2. Kxx
3. KJx

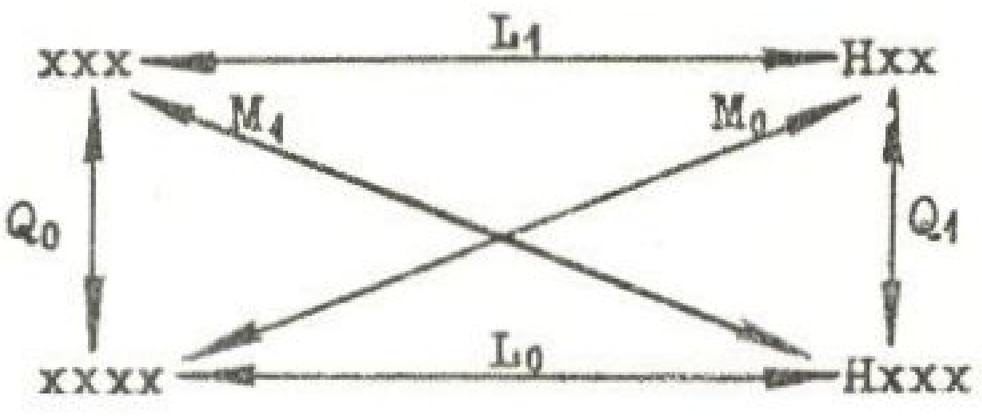
West underled his ace against no-trumps, and South played the queen from dummy which held the trick. When West gets in again, he should lead the ace in case 1; look for an entry to partner's hand in case 2; lead any card in case 3. West will only be able to make the right decision when East's small card is a mixed signal:

even number of small cards = Jxxxx or xxxx
odd number of small cards = Jxxx

Finally, when the quality of any of declarer's suits is known, the mixed signal in that suit becomes a simple length signal; similarly, when declarer's length in any suit is known, the mixed signal in that suit becomes a simple quality signal.

Evaluating signals

Let us evaluate the three types of signals by looking at the amount of information they transmit. Once again, here are the 4 basic possibilities:



Each signal means that partner has two alternatives, and the chance of distinguishing between them is obviously greater when the difference between them is greater. So we have to examine the difference in each of the 6 signals ($L_0\,L_1\,Q_0\,Q_1\,M_0\,M_1$) and arrange them in order of ease of distinguishing between the two alternatives. In this way we will be able to evaluate each of these signals.

The Quality signal (Q) gives partner these alternatives:

Q_0 = xxxx or xxx
Q_1 = Hxxx or Hxx

Both cases are the same, i.e. a small card disappears (or is added) which means that for the purpose of ease of distinguishing both signals are the same, so:

$Q_0 = Q_1$

The Length signal (L) gives partner these alternatives:

L_0 = Hxxx or xxxx
L_1 = Hxx or xxx

Both cases are the same; an honour changes to a small card, or vice versa, which means that:

$L_0 = L_1$

The Mixed signal (M) gives partner these alternatives:

M_0 = Hxx or xxxx
M_1 = Hxxx or xxx

In these cases:

M_0 = an honour changes to two small cards
M_1 = an honour vanishes

So we have 4 types of difference:

Q = a small card vanishes
L = an honour changes to a small card
M_0 = an honour changes to two small cards
M_1 = an honour vanishes

To describe these differences numerically, we must establish the relative values of an honour and a small card. Let us assume that 1½ small cards ≤ honour ≤ 2 small cards, which is borne out by practical experience, backed by calculations, in that trump support of xxxx is more or less equivalent to HHx or Hxx. So we have:

value of a small card = 1
value of an honour = $1 + \delta$ where $\frac{1}{2} \leq \delta \leq 1$

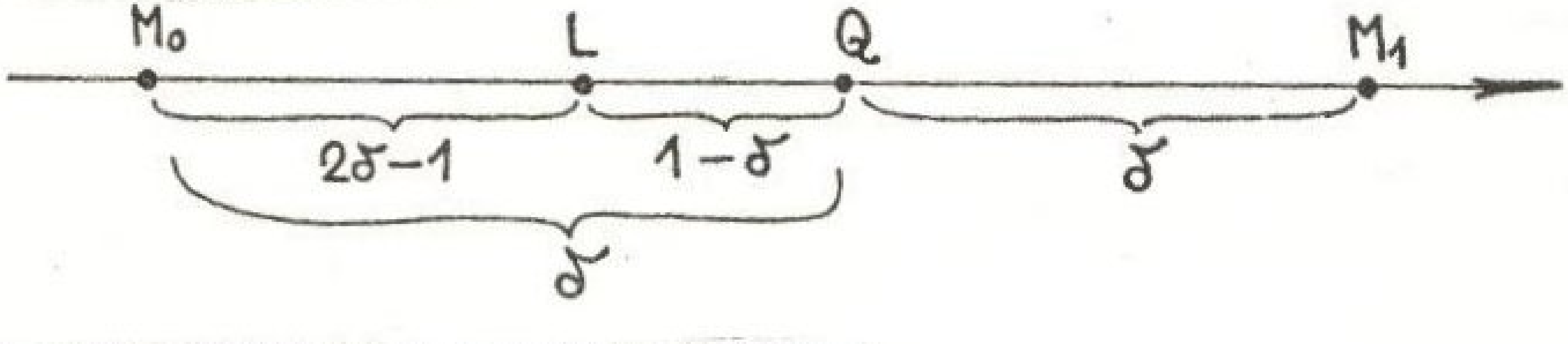
which means that the differences are as follows:

Q = 1 (a small card vanishes)
L = δ (an honour changes to a small card)
M_0 = $1-\delta$ (2 small cards change to an honour)
M_1 = $1+\delta$ (an honour vanishes)

The greater the difference between two alternatives, the greater the information value of the signal. As $1-\delta \leq \delta \leq 1 < 1+\delta$, the order of value of the signals is $M_0 \leq L \leq Q < M_1$

From this we can see that, in spite of popular opinion, length signals are by no means better than quality signals.

Here is a graphical representation:



Let us now try to convert this to percentages. Given that:

1) $\delta = \frac{3}{4}$ (the most likely value)
2) The worst signal (M_0) has a value of 50%
3) The best signal (M_1) has a value of 68%
4) The value of a signal is proportional to the value of the difference between the alternatives

We get:

$M_0 = 50\%$ $Q = 56\%$ $L = 59\%$ $M_1 = 68\%$

SOURCES OF INFORMATION

As we now know approximately what information we are going to impart, we have to discuss the method of imparting this information.

The first two tricks

Information will be transmitted only on the first two tricks in a suit, as by the third trick the position will usually be clear. These two tricks need not be consecutive; they may be separated by one or more tricks in the remaining suits.

The key to signalling with small cards

This cannot be based on attributing specific information to a specific small card, for example we cannot say that the lead of the 3 means Hxx or the lead of the 7 is xxx or xxxx, etc., for the simple reason that the required small card may not be held. The only sensible method is to arrange the small cards in order of rank. Having three small cards, irrespective of their rank, there will always be a highest one, a middle one and a lowest one. So we could agree that from xxx we will lead the highest, from Hxxx the middle, from Hxxxx the lowest, etc.

A notation for the key

To simplify the description of the method we will assume that small cards are written in order of rank from left to right, i.e.:

Hxx is H73 or H62 or H85 etc.
xx is 64 or 52 or 43 etc.
Hxxx is H642 or H853 or H954 etc.

The small card led to the first trick will be denoted by a circle round it and an arrow will lead to the one played to the second trick. For example:

$x\ \overset{\curvearrowright}{\textcircled{x}\ x}$ means that having three small cards we first play the middle one, followed by the lowest one.

$H\ x\ \overset{\curvearrowleft}{x\ \textcircled{x}}$ means that having honour to four we first play the lowest one, followed by the middle one.

A formal definition of a small-card system

So we see that, from a formal point of view, a small-card system is the agreement of a specific method of playing all possible combinations:

xx Hxx xxx Hxxx xxxx etc.

Alternative plays

All the signals discussed in the preceding chapter gave information about one of two alternatives:

Length = even or odd number of cards
Quality = good or bad suit
Mixed = even or odd number of small cards

Let us now consider how we can signal to distinguish between any of these alternatives. If we wish to do this on the first round of a suit, then the best method is to play either the highest available small card or the lowest available small card. However, if we decide to transmit this information on the first two rounds of the suit, it is best to do so by playing small cards in either ascending or descending order, where ascending order means that the small card played to the second trick is higher in rank than the small card played to the first trick, and descending order the reverse.

A notation for alternatives

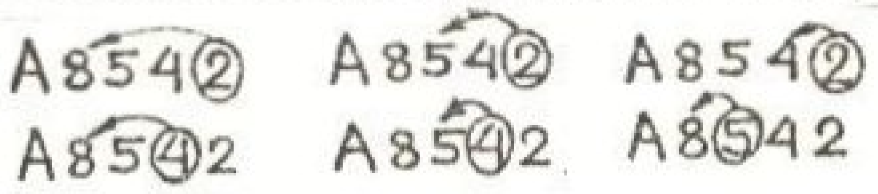
In the description of alternatives we shall use these symbols:-

↑ = playing the highest available small card
↓ = playing the lowest available small card
↶ = playing small cards in ascending order
↷ = playing small cards in descending order

Examples:-

Playing ↑ in relation to the holding Q75 means playing the 7
Playing ↓ in relation to the holding 982 means playing the 2
Playing ↷ in relation to the holding Hxxx means playing either

Playing ↶ in relation to the holding A8542 means playing either



These symbols are very clear and easy to remember.

Sources of information about alternatives

Information about alternatives (either..........or) can come from one of the following sources:-

Source F = First small card
either the highest (↑)
or the lowest (↓)
Source O = Order of small cards
either ascending (↶)
or descending (↷)
Source S = Second small card
either the highest (↑)
or the lowest (↓)

It is vital to remember that the small card itself is unimportant, and that its rank is the transmitter of information.

Assessment of sources

Let us assess briefly the three existing sources of information:

Source F (first small card)

Information is quick but unreliable, as it is often not clear whether partner has played ↑ or ↓.

Source O (order of small cards)

Information is reliable but slow (not clear until the second trick). However, it may be possible to work out whether the first small card is of the type ↑ or ↓.

Source S (second small card)

Apparently the worst, as this information is both unreliable and slow. The situation is not as bad as it might be in that by the second trick a substantial number of small cards will have been played.

As you can see, none of the sources is ideal:

F is quick but unreliable
O is reliable but slow
S is both unreliable and slow

TRANSMITTING SIGNALS

Every signal (L, Q or M) transmits information about one of two mutually exclusive occurrences – A and B. For example:-

A = even number of cards
B = odd number of cards

Every source (F, O or S) transmits information by one of two different ways:

↑ or ↓ for sources F and S
↶ or ↷ for source O

In order to transmit a signal via a source, we must of course decide which of the two occurrences will correspond to which signal. For instance, we could say that:

for source F or S	for source O
↑ means A	↶ means A
↓ means B	↷ means B

Equally, we could say the opposite, and theoretically it would make no difference which method we decided to use. So it follows that every signal (irrespective of the source used) can be used in two ways, one of which we will call the normal signal, the other the reverse signal.

Normal signals (classical)

Normal Length Signal (L)
↓ or ↶ = odd number of cards
↑ or ↷ = even number of cards
Normal Quality Signal (Q)
↓ or ↶ = good quality (an honour)
↑ or ↷ = bad quality (small cards only)
Normal Mixed Signal (M)
↓ or ↶ = even number of small cards
↑ or ↷ = odd number of small cards

All of the above methods of signalling will be referred to as normal or classical and denoted by the symbols L Q M . The methods L and Q have been known and used for a long time, which justifies calling them "classical". Method M, being a new one, has no long history, but one has to start somewhere.

Reverse signals

These differ from normal signals because their meanings are reversed:

Reverse Length Signal (L*)
↓ or ↶ = even number of cards
↑ or ↷ = odd number of cards
Reverse Quality Signal (Q*)
↓ or ↶ = bad quality (small cards only)
↑ or ↷ = good quality (an honour)
Reverse Mixed Signal (M*)
↓ or ↶ = odd number of small cards
↑ or ↷ = even number of small cards

Reverse signals will be denoted by a dot after the symbol for the signal: L* Q* M* .

Normal or reverse ?

Theoretically, both variations of the same signal should be equivalent. In practice, however, the normal signal is better; this is because when you hold, for example, Hxx, the higher small card is often a working small card, so it is best not to have to play it to the first trick in order to signal.

Which quality signal is classical ?

The normal quality signal (↓or↶= good quality) is applied as follows:

⊗x
⊗xx Hx⊗
⊗xxx Hxx⊗ etc.

or: low small card = good suit
high small card = bad suit

It may come as a surprise that this method is called "normal" by the author, as popular nomenclature would call it "reverse". However, it is worth noting that it is equivalent to Culbertson's small-card system, where you lead:

from small cards — your highest small card
from an honour — your lowest small card

Thus the method "↓or↶= good quality" is more classical than the opposite (↑or↷= good quality) and deserves to be called "normal". That this is not the case is the result of a misunderstanding.

The reason for the misunderstanding — encouragement

To understand why "↑ = good quality" has been given the name "normal", let us assume that you are defending in classical Culbertson style and partner has led the king (showing AK or KQ) against a suit contract. Which small card should you play from the following holdings:

XX QXX XXX

Since you are leading in classical style, it would be most convenient to use the same signals when following suit as when leading, i.e. low small card = good quality, which means you would play thus:

⊗X QX⊗ ⊗XX

However, this is not a good situation to be in as partner will not be able to distinguish between two and three small cards. So perhaps it is better to use the normal length signal (low small card = odd number of cards):

⊗X QX⊗ XX⊗

This is also bad, because now partner will not be able to differentiate between three small and queen to three. So in this type of situation neither normal quality signals nor normal length signals are useful (the same applies to reverse signals). To avoid this, encouragement was introduced; a small card played to a trick which partner has led to means:

— continue the suit (encouragement)
— switch (discouragement)

As it is instinctive to assume that high = positive and low = negative, the following system of encouraging was used:-

↑= encouraging
↓= discouraging

which, after a long period of use, came to be called normal (classical). In turn, because it is more common to encourage with an honour than with small cards only, "↑" came to mean "good quality". This was termed normal in spite of the fact that the classical small-card system is based on "↑= bad quality". One final point; "↓ = encouraging" is better, as when you possess an honour, a high small card in that suit is more often a working small card than in a weak suit. So this method should be termed normal.

DEFINITION OF A SMALL–CARD SYSTEM

There are two ways of defining a small-card system:
-- either strictly formal
-- or structural

Formal definition

Operates by agreeing which order small cards should be played (at both the first and second tricks) for every particular holding:

xx	
xxx	Hxx
xxxx	Hxxx
xxxxx	Hxxxx
xxxxxx	Hxxxxx

For example:

etc.

This definition enables you to describe any random small-card system, i.e. it is universal. However, it is uninteresting in that it does not delve into the meaning of the cards played, limiting itself to a formal description.

Structural definition

This describes a small-card system as the interaction of three sources of information, each of which transmits a given signal. We have three sources at our disposal: F O S , and six possible signals:

	Normal	Reverse
Length	L	L*
Quality	Q	Q*
Mixed	M	M*

To define a small-card system we have to establish which signal will be transmitted through which source. This means that a small-card system can be described as tripartite: S_F S_O S_S

where:
S_F = signal transmitted by F
S_O = signal transmitted by O
S_S = signal transmitted by S

For example: LQM QQL* ML*Q Q*MM QL*M M*M*L

Interaction of sources

At first sight it might appear that each source could transmit any signal, irrespective of which signal the others transmit. However, it would be pointless to have the same ambiguous information transmitted by two sources, when one uses a normal signal and the other a reverse signal. If they transmit the same information, they should do so using the same version of the signal. The amount of information will not become less, and the signalling method will be more concise.

Conclusion: Only one of two opposite signals should be used.

Equally, there is no point in transmitting the same information through sources O and S, as both sources give the information at the same time (the second trick), and source O gives accurate information. So $S_0 \neq S_S$. On the other hand, sources F and O can be used to transmit the same information. This ensures both speed (F) and accuracy (O). Also, this interaction of F and O means that it is often easier to determine whether the card played to the first trick was of the type ↑ or ↓.

Nomenclature of classifiable systems

The symbolic name of a system which can be described using the structural definition (i.e. classifiable) is S_F S_O S_S

Systems in which the lowest possible card is played at trick two will be denoted by S_F S_O↓or simply S_F S_O

Each source will be assigned a specific signal, with the proviso that the signal corresponding to S will be either in brackets or omitted.

Some examples:

QML = quality -- mixed (length)
QL*↓ = quality -- reverse length
L*M = reverse length -- mixed

In systems where S_F = S_O , the first two signals will be the same, for example:

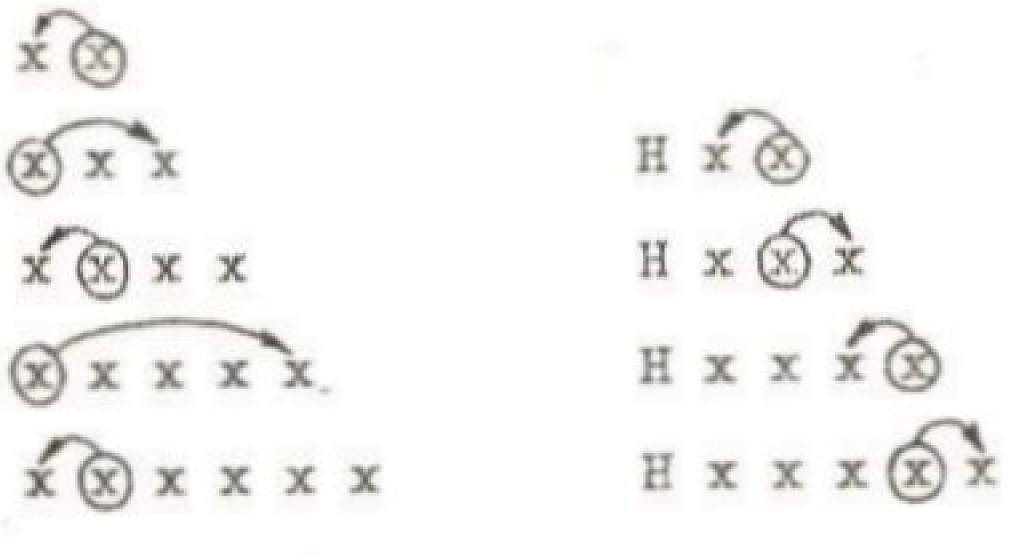
QQM = quality (mixed)
MM↓ = mixed
L*L*Q = reverse length (quality)

Reconstruction of a system using its name

Taking the small-card system QM:

Source F : transmits signal Q, or
↓= good suit (an honour)
↑= bad suit (no honour)
Source O : transmits signal M, or
↶ = even number of small cards
↷ = odd number of small cards
Source S : ↓ (as there is no third symbol in the name)

Thus, using the system QM, you would play:-



Note that ↓ is not always the smallest card. To be more exact, it is the smallest card consistent with the signal transmitted by source O. This also applies to ↑.

VARIOUS SMALL–CARD SYSTEMS

In this chapter we shall describe various small-card systems. They fall into three categories:-

1) Traditional or popular systems
2) Classifiable systems
3) The "Combine" system

Traditional or popular systems are in general unclassifiable. They are:-

Classical	CLA
MUD	MUD
Reverse	REV
Blue Team	BT
Journalist	JOU

As some of these systems are incomplete, i.e. it is not clear which card should be played from certain holdings, the author has had to reconstruct them, at the same time making a few small corrections relating to source S. Classifiable systems are, on the whole, completely new, having no correspondence with traditional methods. The Combine system has been singled out as it is as yet little known, as well as being unclassifiable.

The Classical system CLA

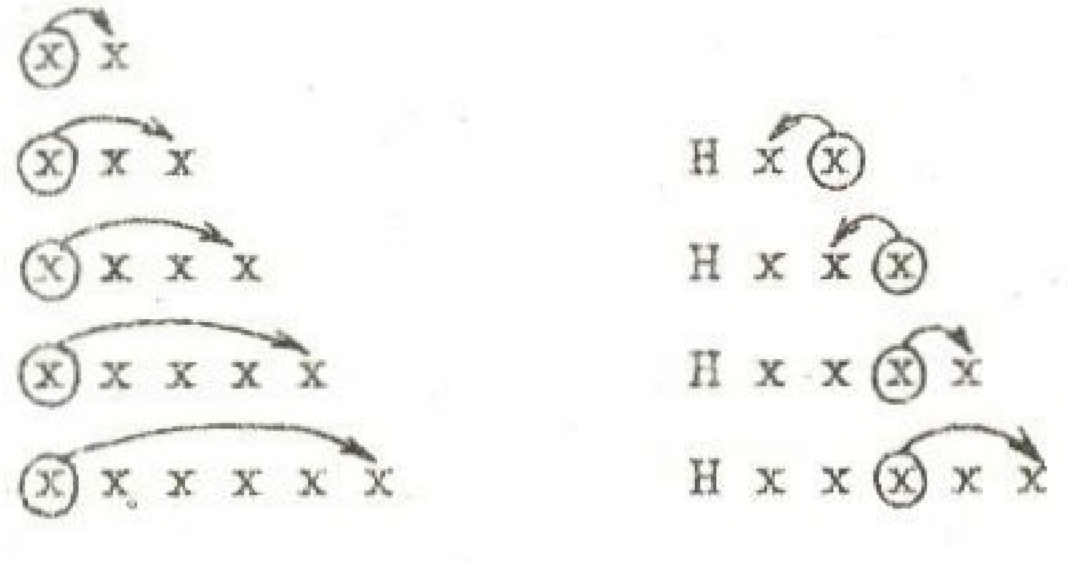
This is oriented towards swift information (at the first trick) as to suit quality:

↑= bad suit (small cards only)
↓ = good suit (including an honour)

Or, to put it in more scientific terms, source F transmits a normal quality signal. Source O confirms the information transmitted by F as it also transmits a normal quality signal:

↷= bad quality
↶ = good quality

The exceptions to the above rules are when you hold honour to five or six. If it were not for them, the Classical system could be classified as quality (QQ↓).



The "fourth highest" convention

The symmetry of the Classical system is broken when the holding is honour to five or six. Then the following rule applies:- From a good suit, i.e. at least four cards in length and including an honour, lead the fourth highest card from the top. The advantages of this method are:-

1) It gives partner some idea of suit length, as:
   - playing a higher card to the second trick indicates Hxx or Hxxx
   - playing a lower card to the second trick indicates Hxxxxx or Hxxxx
2) Partner knows that you possess three cards higher than the one led. So in this position:-

Q653

AJ98 K102

when West leads the 8, East can confidently play low as long as he knows that West has at least four cards in the suit.

The MUD system MUD

The Classical system has the drawback that it is often impossible to distinguish between two and three small cards, even after two tricks. To circumvent this, the MUD system was invented, where with three small cards the middle one is led, followed by the highest one. Schematically:-

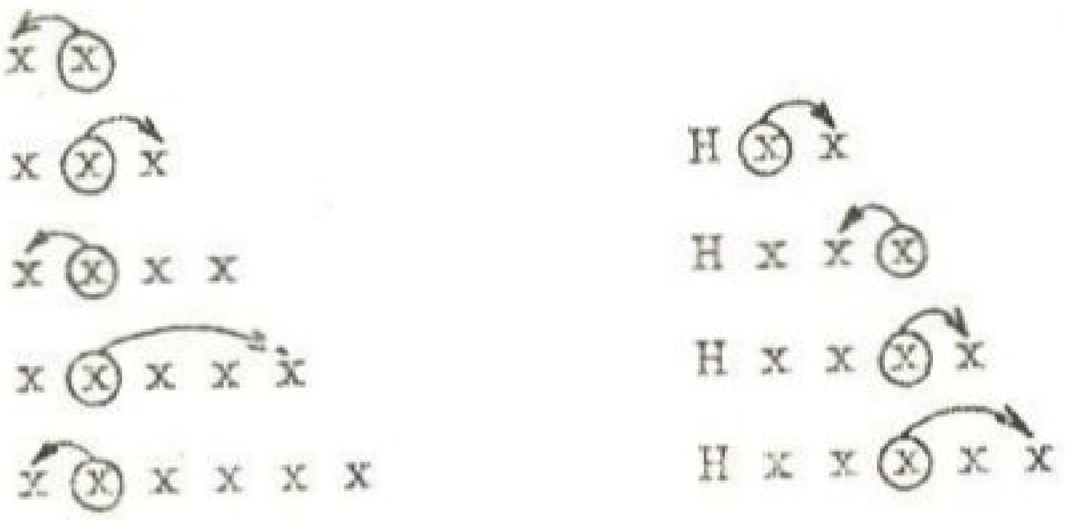
The name is a mnemonic, standing for:-

M = Middle
U = Up
D = Down

As a result it is possible to distinguish between "xx" and "xxx", also between "xxx" and "xxxx" (but generally only after the second trick). When it comes to distinguishing between "xxxx" and "xxxxx", and between "xxxxx" and "xxxxxx", there is no information available, so it seems sensible to play xⓧxxx.

The Reverse System REV

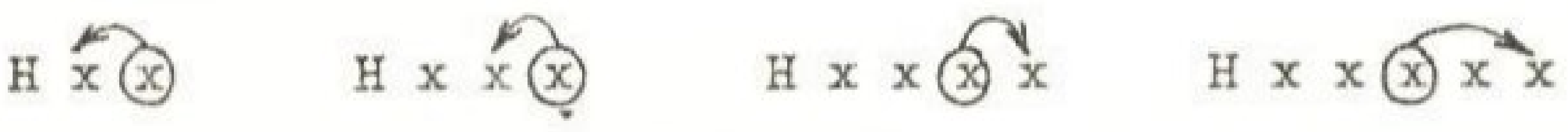
The Reverse System differs almost totally from the classical style, introducing new and original ideas:



One of its characteristics is that the highest card is never led, which means that if all the cards higher than the one led can be seen, you can be certain that the lead is a singleton ! This is not as useful as it appears, as the situation only arises very infrequently. A more significant feature of not leading the highest card is that partner knows you have precisely one card higher than the one led (except when the lead was fourth highest). A big drawback of the Reverse System is its inability to distinguish between xxx and Hxx, even after two tricks.

The "third and fifth highest" convention

Fourth-highest leads in the Classical and MUD systems:

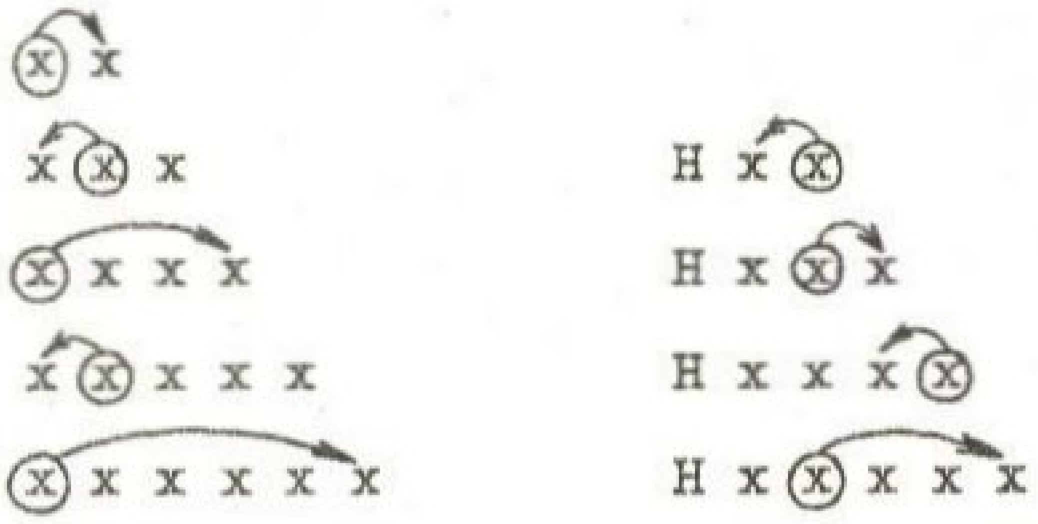


meant that while you could distinguish between honour to four and honour to five, it was often impossible to tell whether partner had Hxx or Hxxx, and Hxxxx or Hxxxxx. An effective remedy appeared in the form of third and fifth highest leads:

In this way it was possible to tell one holding from another, at the same time retaining the classical rule " ↓ = good suit". It also meant that instead of there being three cards higher than the one led in partner's hand (with 4th highest leads), the lead of the third highest marked partner with only two cards higher than the one led, which makes decisions of the type "go in with an honour or duck" significantly easier.

The Blue Team System BT

This is a modification of the MUD system based on replacing "fourth-highest" with "third and fifth":



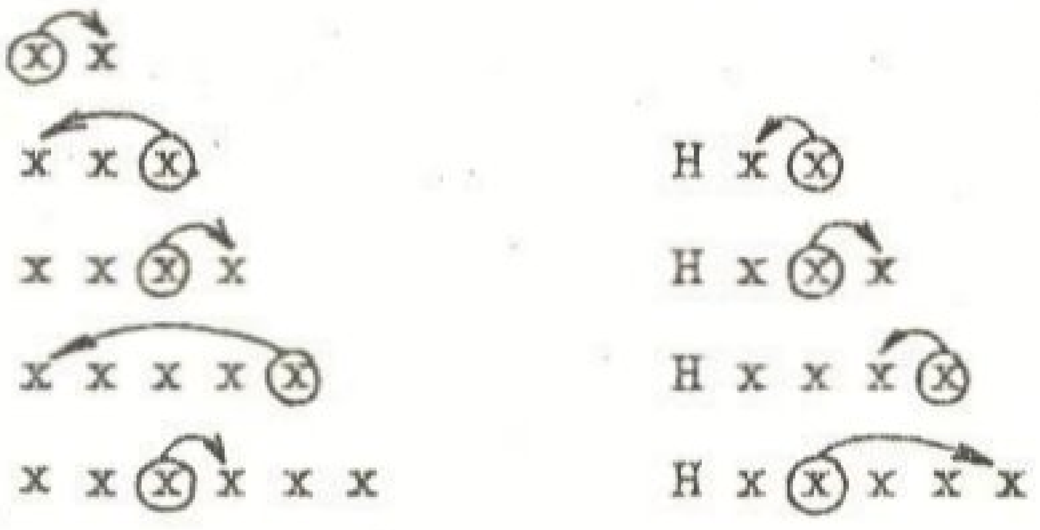
The Blue Team System shows a for-reaching logic and symmetry:

Source F – transmits signal Q
Source O – transmits signal L
Source S – is equivalent to ↓

The only irregularity is Hxⓧxx. If this were replaced by Hxxⓧx then the BT system would be identical to the classifiable system QL (quality–length). The conclusion we can draw from this is that the Blue Team System is the last rung on the ladder of evolution of the Classical System.

The Journalist System JOU

Against no-trump contracts this is almost identical to the QQ (quality) system. But against suit contracts the leads are:



This, as you can see, is the application of third and fifth leads to suits of both good and bad quality. From a practical point of view it is similar to the system ↓LQ.

N.B. In the source text no mention was made of the card played to the second trick (assumed to be ↓). The author has assumed that, wherever possible, source S transmits signal Q.

Classifiable Systems

Now we can elaborate on the various classifiable systems, i.e. those which can be described using the structural definition. They consist of two groups:

1) Systems where $S_F = S_O$, or

$\left.\begin{array}{l}QQL\\QQM\end{array}\right\}$ group QQ $\quad$ $\left.\begin{array}{l}LLQ\\LLM\end{array}\right\}$ group LL $\quad$ $\left.\begin{array}{l}MML\\MMQ\end{array}\right\}$ group MM

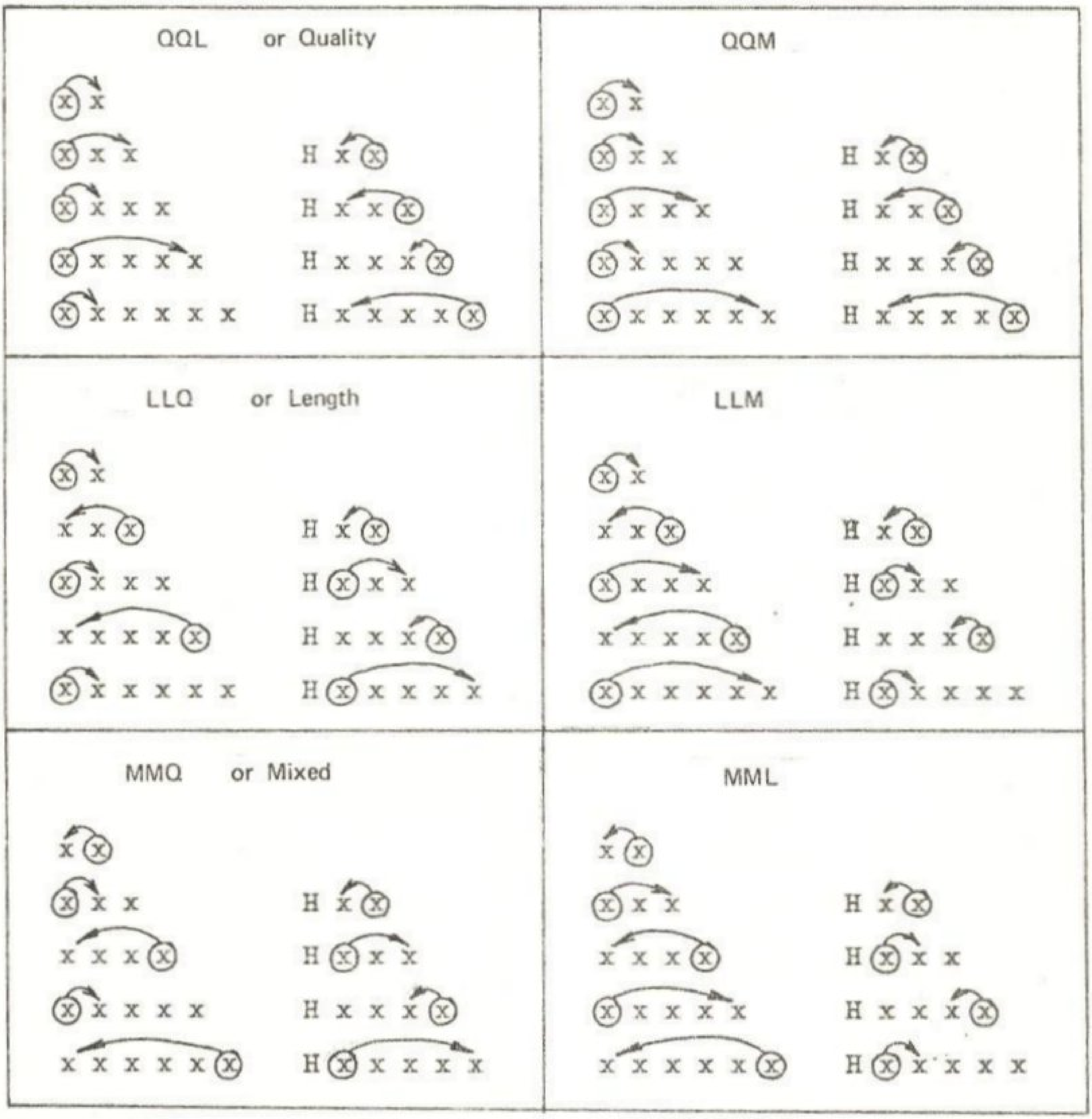
2) Systems where $S_F \neq S_O$, or

$\left.\begin{array}{l}QLQ\\QLM\end{array}\right\}$ group QL $\quad$ $\left.\begin{array}{l}QMQ\\QML\end{array}\right\}$ group QM

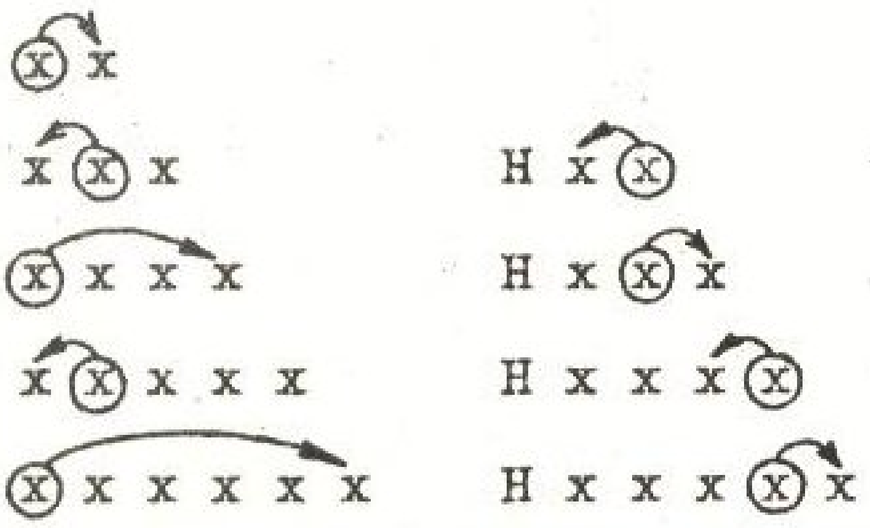
$\left.\begin{array}{l}LQL\\LQM\end{array}\right\}$ group LQ $\quad$ $\left.\begin{array}{l}LML\\LMQ\end{array}\right\}$ group LM

$\left.\begin{array}{l}MQL\\MQM\end{array}\right\}$ group MQ $\quad$ $\left.\begin{array}{l}MLM\\MLQ\end{array}\right\}$ group ML

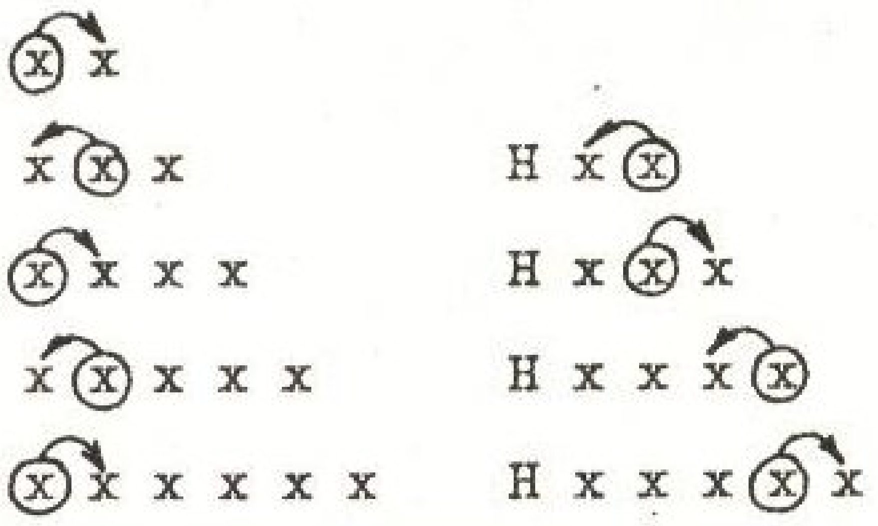
As you can see from the above, only the normal version of any signal has been used; the fact that every reverse signal is exactly equivalent to its normal version justifies this.



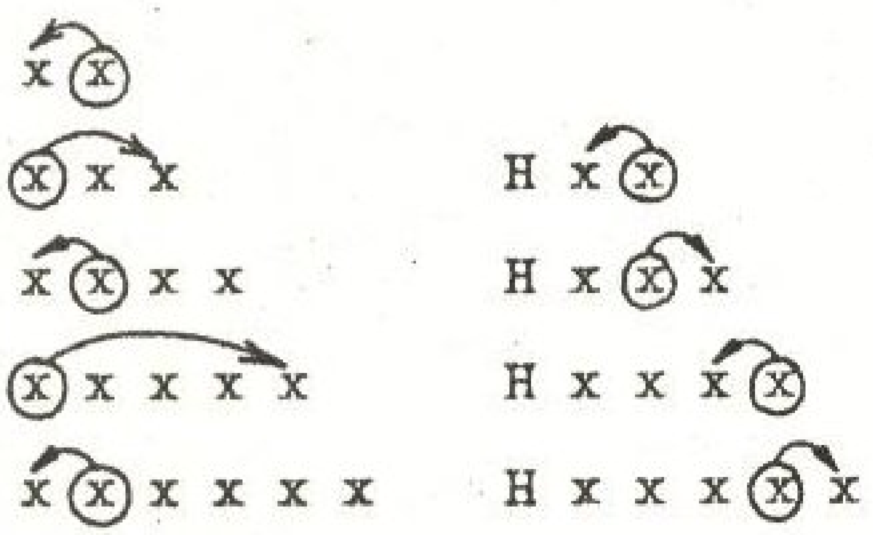
QL = QLM or Quality-length



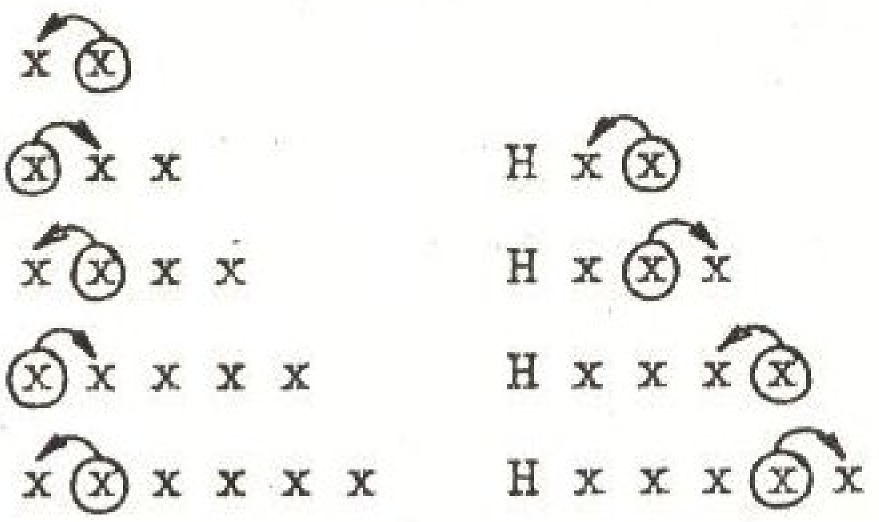
QLQ



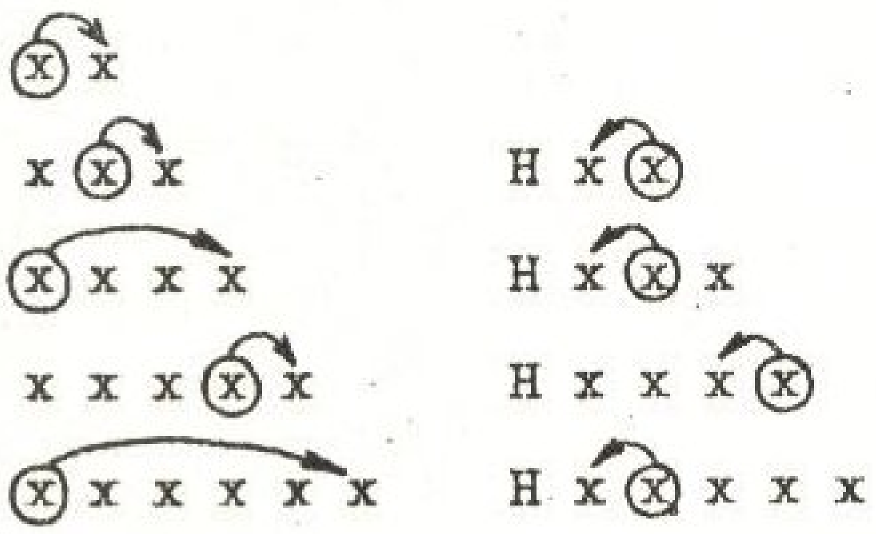
QM = QML or Quality-mixed



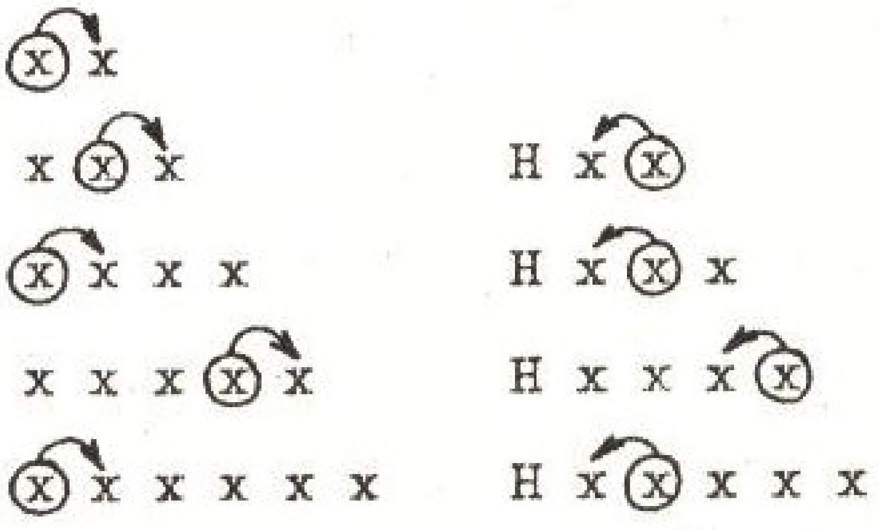
QMQ



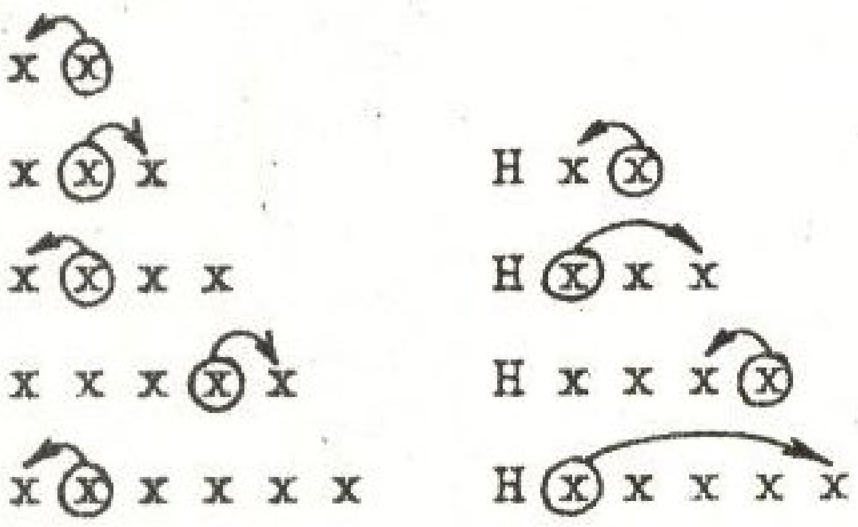
LQ = LQM or Length-quality



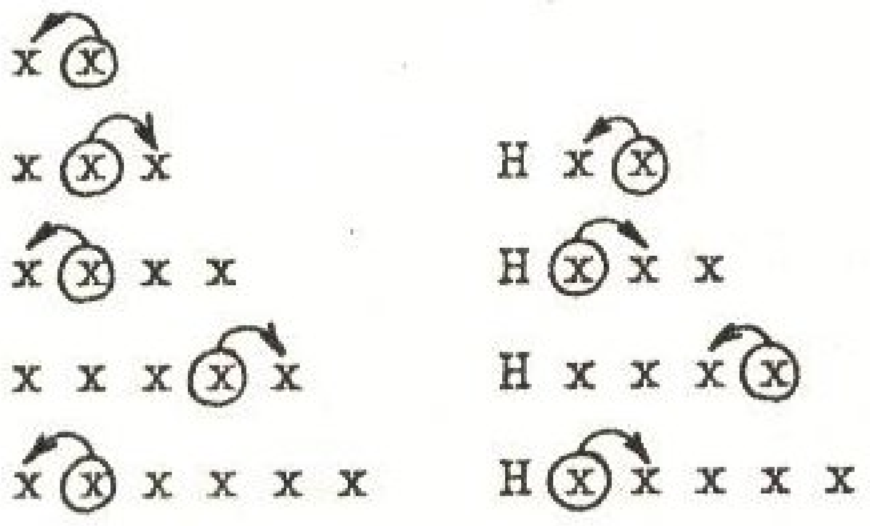
LQL

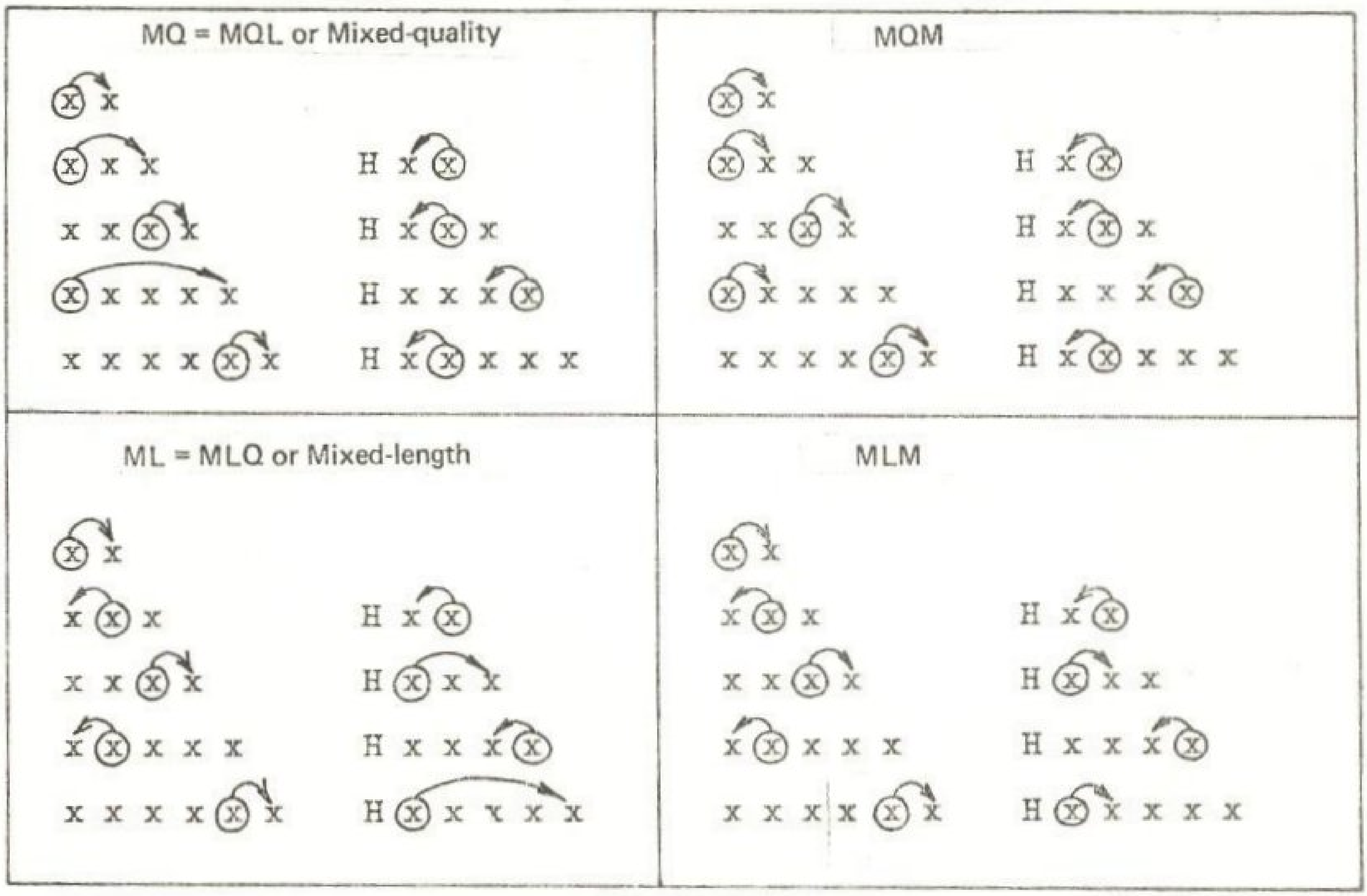


LM = LMQ or Length-mixed

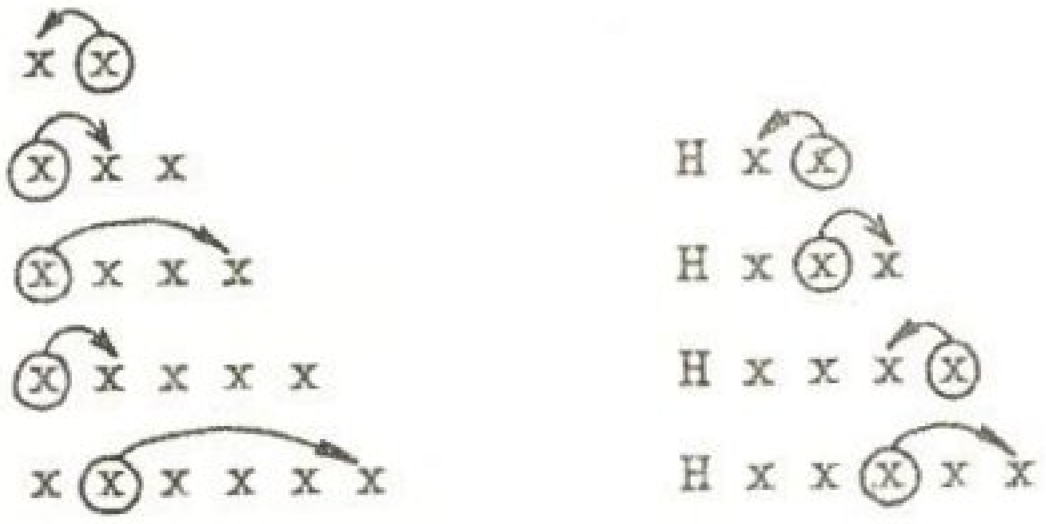


LML





The Combine System C



Even though Combine is unclassifiable, it does have a certain regularity:
Source F : ↓ = attacking lead (from an honour or a doubleton)
↑ = passive lead (from xxx, etc.)
Leads from three or more small cards : as in QQL (slightly different ffom xxxxxx)
Leads from an honour : as in QL (slightly different from Hxxxxx)
An exact description of Combine can be found in Part Three.

PERMUTATIONS OF SMALL CARDS

Some readers may be of the opinion that the foregoing theory of small-card systems is not comprehensive enough, as it does not embrace such ideas as odd-even signals, for instance. These are based on:

odd small card = odd number of cards
even small card = even number of cards

However, we shall show that this is not the case, with some interesting consequences.

Odd-even Signals

These mean that:
having an even number of small cards, we play an even small card (2468)
having an odd number of small cards, we play an odd small card (3579)

But what if we have an even number of cards, all of which are odd ? (Or vice versa ?) In that case, we must also take into account the rank of the small card, and agree that, for example:

$$\left.\begin{array}{l}\uparrow = \text{even number of cards}\\ \downarrow = \text{odd number of cards}\end{array}\right\}\text{ signal L}$$

Next, we must ask ourselves: Having an even number of cards, can we lead any even one (assuming we have more than one) ? Similarly, having an odd number of cards, can we lead any odd one ? Obviously it would be wasteful to ignore any possibility of signalling, so we should use some length signal, say L (normal length). So this version of odd-even signals is in principle the system LL (length) with the following restriction:

If possible the first card played should be even when holding an even number of cards, and odd when holding an odd number of cards.

It is an easy matter for the reader to determine that these are simply ordinary length signals, with the proviso that the rank of small cards is 86429753, or $8 > 6 > 4 > 2 > 9 > 7 > 5 > 3$.

Applying the signal L (normal length) to this order of small cards, we see that:

Holding an even number of cards we have to play an even small card (as they are "higher" than odd ones)

Holding an odd number of cards we have to play an odd small card (as they are "lower" than even ones)

Lacking a suitable small card (or having a surplus of them) the signal L given will be identical to that given when the order of small cards is "normal", as $8 > 6 > 4 > 2$ and $9 > 7 > 5 > 3$.

Odd-even signals can, of course, be used in several versions (various combinations of L and L*) which correspond to the following order of small cards:

86429753 97538642 35792468 24683579
86423579 97532468 35798642 24689753

Theoretically, all these versions are completely equivalent. Clearly, they are also as good as normal length signals (98765432) or reverse length signals (23456789).

Permutations of small cards

The case of odd-even signals demonstrates that there is no need to slavishly adhere to the natural order of small cards: 98765432. The numbers on small cards serve solely to distinguish between them, or, more exactly, to compare their rank. To this end, however, we can use any random order of small cards, for example: 72693548 93726458 24589763 etc.

Using any given order of small cards, we can play any small-card system, for example:

"We lead classically, but the order of small cards is 49732856"

"We use normal length signals, but small cards rank as follows: $3 > 6 > 5 > 9 > 4 > 7 > 8 > 2$"

"We play Combine, but the small card x is higher than the small card y if and only if $|\cos(x)| < |\cos(y)|$"

Obviously, these things have no intrinsic merit, their only good point being that they demand a lot of concentration from opponents unfamiliar with them. They can, however, be used, as the laws of Bridge have not, as yet, banned the use of conventions whose sole purpose is to confuse opponents and force them to expend extra mental effort.

Minimisation of the first small card

An idea of Marek Dryański's caused me to reconsider my conclusion as to the intrinsic merit of permutations of small cards. It appears for the first time in this, the third edition of this book:

> For any given small-card system, find a permutation of small cards such that the average rank of the first small card played is as low as possible !

Clearly, the smaller the card played to the first trick, the smaller the chance of it being a working card, thus reducing the risk of losing a trick. Marek Dryanski uses the system MML (mixed) with this order of small cards: 3 5 7 9 10 8 6 4 2 where the lower small cards are positioned at the extremes of the order. This permutation of Dryanski's is excellent for systems using the highest or lowest small cards at the first trick (MM, LL, QQ), although it is not necessarily the best. Finding an optimum order of small cards for other systems would require a lot of work, and the resultant gain would be minimal. Perhaps the fourth edition will have more to say on the matter.

Equivalence of systems

Two small-card systems will be termed equivalent when one of them applied to a given permutation of small cards is identical to the other. For example, when the Reverse System is applied to the permutation 2 9 3 5 7 4 6 8. the result is an equivalent small-card system with, amongst others, these plays:

This apparent lack of regularity disappears when the small cards in the above holdings are written not in their natural order but in the order of the permutation 2 9 3 5 7 4 6 8 :

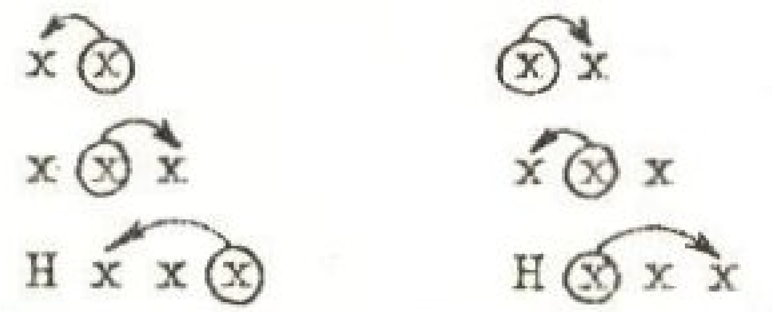
Each class of systems mutually equivalent contains as many systems as there are permutations of nine small cards (i.e. 9! = 3,265,920). Equivalent systems have, of course, the same informative properties and from that point of view are equally good.

Reversed Systems

One specific instance of the equivalence of systems is the equivalence gained by applying the reverse order of small cards (2 3 4 5 6 7 8 9).

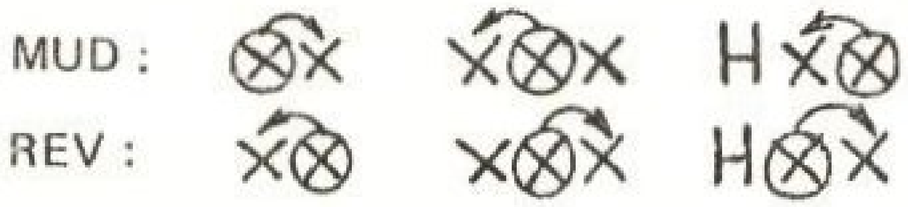
> System A is the reverse of System B when it is created by applying the signals of System B to the reverse of the natural order of small cards, i.e. 2 3 4 5 6 7 8 9.

For example, using normal length signals (↓ = odd) with the order 2 3 4 5 6 7 8 9 is the same as using reverse length signals (↑ = odd) with the natural order, and the same applies to quality and mixed signals. It is especially easy to work out a given play in a reversed system; it is enough to imagine that the small cards are written in reverse order, thus:



........etc.

The fact of equivalence (and especially reversal) saves us considerable effort when researching into small-card systems: with two reversed systems, it is sufficient to analyse one, as the efficiency of the other will be the same. For example, in Problem 2 – 3 the systems below are reversed (and thus equally informative):



Reversed Combine

From an informational point of view, it is as good as normal Combine. In practice, it is somewhat worse, in view of the fact that from an honour a high small card is led (which may be a working small card).

METHODS OF EVALUATING SMALL–CARD SYSTEMS

Thus far the theory of small-card systems has not found the answer to the question – which systems are best and why ? So we have to find a method of assessing the quality of a small-card system. We must dismiss all criteria based on observation or the opinion of even the best players. These are false criteria, biased by years of habit and coloured by the chance effect of some fascinating deal. Equally useless would be citing numerous examples of a method's efficacy. Even were we to cite a hundred examples, a determined critic would give a thousand counter-examples. The only objective method of evaluation can be a statistical method, based on examining every possible significant occurrence and counting successes and failures. Such a method was presented by the author in an article, "Distributional Leads" (Brydż, Sept. 1974) where it was used to determine how to differentiate between a doubleton and a tripleton. In this book it has been extended to apply to length-quality problems.

An example of comparison of small-card systems

Examine this defensive situation:

xxxx

? AJ10

?

Your partner, West, has led a small card against a suit contract. You have won the ace and returned the jack, which declarer won with the king. From the bidding it is clear that South has 2 or 3 cards in the suit (i.e. partner has 4 or 3), which means that there are three possibilities:

xxxx

1) xxx
2) Qxx AJ10
3) Qxxx

1) KQx
2) Kxx
3) Kx

In the hidden hands (partner's and declarer's) there are four small cards. Is it relevant which ones ? No ! They can be any four small cards, as the important thing is solely the rank of small cards. So let us say that they are the four lowest : 5 4 3 2. Now let us check how certain systems perform in this situation. How often, irrespective of which small card declarer has played, will we be certain which holding (xxx, Qxx or Qxxx) partner has ?

West has :	CLA	MUD	REV	BT	JOU	C
543	53 ●	45	43	45	35	54 ●
542	52 ●	45	42	45	25	54 ●
532	52 ●	35	32	35	25	53 ●
432	42 ●	34	32	34	24	43 ●
D54	45 ●	45	54 ●	45	45 ●	45 ●
D53	35 ●	35	53 ●	35	35	35 ●
D52	25 ●	25 ●	52 ●	25 ●	25	25 ●
D43	34	34	43	34	34 ●	34 ●
D42	24	24	42	24 ●	24	24 ●
D32	23	23	32	23 ●	23 ●	23 ●
D543	34	34	34 ●	43 ●	43 ●	43 ●
D542	24	24	24 ●	42 ●	42 ●	42 ●
D532	23	23	23 ●	32 ●	32 ●	32 ●
D432	23	23	23 ●	32 ●	32 ●	32 ●
	7	1	7	7	7	14

The above table lists all possible combinations of small cards in the West hand (14 possibilities). For every small-card system the cards played by West have been given, and the symbol ● signifies that West's holding is known with complete certainty. As you can see, in this problem Combine has a 100% rate of success; this would also apply to the systems QM, LM, and MML.

An example of statistical analysis

We shall now conduct a detailed analysis (using the same defensive problem) of the MUD system.

		First trick			Second trick			
		F	I_0	I_1	S	II_0	II_1	II_2
x ⓧ x	543	4	⊡	-	45	⊡		●
	542	4	⊡		45	⊡		●
	532	3	┐+	□	35	□		●
	432	3	┐+	□	34	┐+	□	●
H x ⓧ	D54	4	□		45	□		●
	D53	3	┐·	△⊡	35	⊡		●
	D52	2	△		25	●		
	D43	3	┐·	△⊡	34	┐·	△⊡	●
	D42	2	△		24	△		●
	D32	2	△		23	△		●
H x x ⓧ	D543	3	┐	△	34	┐	△	●
	D542	2	◬		24	◬		●
	D532	2	◬		23	◬		●
	D432	2	◬		23	◬		●
63.10%	●					1	1	14
	○							
	□		3	5		5	6	
	△		6	9		5	7	
	┐		5			3		
	⊕		13	14		13	14	14
	◍		7	7		8	8	14

Small cards played by partner

F = first card (i.e. played to the first trick)
FS = first and second cards

Informative situations

The desired information depends not only on the cards played by partner, but also on those played by declarer. Bearing this in mind, there are five informative columns in the table ($\mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2$), and each of them are marked with the appropriate symbol relating to the information gained. The columns refer to:

I_0 = information gained at trick 1, assuming declarer has played 0 small card
I_1 = information gained at trick 1, assuming declarer has played 1 small card
II_0 = information gained at trick 2, assuming declarer has played 0 small card
II_1 = information gained at trick 2, assuming declarer has played 1 small card
II_2 = information gained at trick 2, assuming declarer has played 2 small cards

It is worth noting that in a concrete defensive problem, only two of the above situations can occur. For example:

876
? KQ109
A......

Situations I_0 , II_1

876
? AKJ10
?

Situations I_1 , II_2

However, in a theoretical analysis, all situations should be considered, so that the table has a more universal character and relates not only to the example problem but also to other problems of the same type.

Cards played by declarer

Let us assume that declarer plays the optimum cards for him, i.e. those which give us a minimum of information, and never plays an honour unless he has to. This assumption is biased in favour of the declarer, as he will often be either not good enough to play the correct card or simply too lazy to do so. In theory, however, we have every right to assume that the declarer is infallible.

Information

Each of the informative columns ($\mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2$) are marked with the symbol corresponding to the information gained:

• = accurate information (one-way)
o = mixed information ⎫
□ = length information ⎬ two-way
△ = quality information ⎭
⌝ = three-way information

Four-way information (i.e. a total lack of information) has no specific symbol; it is merely denoted by a blank space. Obviously this situation cannot arise in problems of the type 2 – 3, as from Hx the honour would be led.

As we move from right to left along the columns, information from either trick generally increases, and never decreases.

Schematically:
$$\mathrm{I}_0 \leq \mathrm{I}_1$$
$$\mathrm{II}_0 \leq \mathrm{II}_1 \leq \mathrm{II}_2$$

This fact enables us to simplify our symbology:

1) If there is a blank space in any column (except I_0 or II_0) it denotes the same information as in the preceding column.
2) If the symbol • appears in column I_0 then there will be a blank space in all subsequent columns.

To summarise: a blank space denotes either a repeat of previous information or four-way information.

In the example table repetition of information occurs frequently (in columns I_1 , II_1), but there is no case of four-way information as we have not considered the possibility of the lead being from xxxx, as it is only an example table.

It may also be the case that declarer has at his disposal two optimum cards, and, depending on which one he plays, we will have different information. This situation will be denoted by a double symbol (e.g. column I_1 , line Q43).

Summary indicators

Totals of the various kinds of information are given at the bottom of the table in the lines marked with the symbols • □ o △ ⌝ . For example, in the column II_0 :

1 case of • (certainty)
0 cases of o (two-way mixed)
5 cases of □ (two-way length)
5 cases of △ (two-way quality)
3 cases of ⌝ (three-way)

Success indicators ⊕ ⊙

Summary indicators enable us to compare systems fairly well. But there are dubious situations where it would be better to use one figure rather than several summary indicators to describe the efficiency of a system in every informative situation. For a measure of quality we shall use the success indicators, which indicate how often we can make the correct decision on the basis of available information. These indicators (denoted by the symbols ⊕ ⊙) appear at the bottom of the table, and their exact meanings are:-

⊕ = number of correct decisions assuming two-way information is sufficient
⊙ = number of correct decisions assuming one-way information is needed

A correct decision will be denoted by inserting an extra symbol in the symbols ○□△⅂ or "blank":

a cross (+) for indicator ⊕

a dot (•) for indicator ⊙

When there is two-, three- or four-way information decisions will be based on the assumption that all possible distributions of small cards have an equal probability. For example, in the example table there are 14 equally probable possibilities: 543, 542,........, Q54, Q53,........Q543, Q542,........

Obviously, this assumption is a simplification of the true probabilities, but the error introduced by it is not significant, and the analysis can be done without the use of a computer.

Example: calculation of ⊙ in II_o

A correct decision will occur only when partner has the particular holding (i.e. xxx, Qxx or Qxxx) we play him for, i.e. the most likely holding based on available information. Let us see how that works depending on the cards played by partner:

45 Partner can have 543, 542 or Q54, which means we play him for xxx (odds of 2 – 1). We will be correct when he has 543 or 542, so we put a dot in the symbol □ on the appropriate lines.

34 Partner can have 432, Q43 or Q543, which means we have three-way information (xxx, Qxx or Qxxx). As all three possibilities are equally likely, we can play him for any of them – say Qxx. So we put a dot in the symbol □ on line Q43.

and so on. Altogether it will turn out that in situation II_o we get 8 correct out of 14.

Example: calculation of ⊕ in I_o

This time, a correct decision will occur when partner has the more likely of the two possibilities when we have two-way information. Thus all correct decisions of the type ⊙ are included in ⊕, which means we only need to consider three-way and four-way information.

In situation I_o , the only time three-way information occurs is when partner leads the three, which means:-

either 532 ,432 (xxx)
or Q53 ,Q43 (Qxx)
or Q543 (Qxxx)

So we should play him for the two-way "xxx or Qxx", which means we will be correct in four cases:
532 432 Q53 Q43.

Correct decisions 532 432 are denoted by a cross inserted in the symbol ⅂. Correct decisions Q53 Q43 need not be denoted by a cross as they have already been marked with a dot in the calculation of ⊙ (and each ⊙ is included in ⊕). So we see that the indicator ⊕ is based on the assumption that two-way information is sufficient for a correct decision.

Efficiency of a system

In the bottom left-hand corner of the table is a percentage, which describes the overall efficiency of the system for that problem, or, more simply, the percentage of correct decisions. This is calculated as follows:

1) Calculate the average of correct decisions of the type ⊙ using the following weightings: $\mathrm{I}_o = 7/18$ $\mathrm{I}_1 = 5/18$ $\mathrm{II}_o = 2/18$ $\mathrm{II}_1 = 3/18$ $\mathrm{II}_2 = 1/18$
2) Repeat the procedure for ⊕
3) The average number of total correct decisions is 8/18 average ⊙ + 2/10 average ⊕
4) The resultant number is divided by the number of possible distributions and expressed as a percentage.

This answers the question: How often will we correctly decide what partner has led from ?

Equal two-way information

This occurs when we have to choose between two alternatives: partner is known to have either A or B, and both are equally likely.

Some examples:

Type 1 : 1 = H54 or 654
We can choose either Hxx or xxx, as the chances are equal (as all possibilities have an equal probability) and are 1/2 = 50%.

Type 2 : 2 = H54 H53 or 654 653
We can choose Hxx or xxx, as both have an equal probability of 2/4 = 50%.

Type 3 : 3 = H54 H53 H52 or 654 653 652
We can choose Hxx or xxx, both having an equal probability of 3/6 = 50%.

In general: we may be dealing with equal two-way information of the type n : n.

Counting successes of type ⊙ when dealing with equal information

If we were to rely solely on the equal probability of all possibilities, then when dealing with equal information of the type n : n the success rate would be 50% (n successes out of 2n occurrences). But we know that apart from the information given to us by the small-card system we are in possession of other information (from the bidding and play), thanks to which the chance of resolving equal information rises to over 50%. This has been analysed in "Evaluation of signals", coming to the conclusion that the chance of resolving equal information depends on the type of signal, the percentages being:

50% for signal M_0
56% for signal L
59% for signal Q
68% for signal M_1

Thus the success rate when dealing with equal information of the type n : n is the same as the percentages above. However, in the previous edition of this book, I assigned the following (incorrect) values:

L = Q = 50% $M_0 = M_1 = 67\%$

As a complete correction of the above error would require a vast amount of work, I have rectified it in the following manner:

1) For L and Q the value remains at 50%. As they are almost equivalent the error introduced by this is very small.
2) M_0 becomes less than 50%. To be more exact, the value for M_0 is reduced by as much as it has been increased for M_1 . This works because the ambiguity of M_0 is always linked with M_1 (in the same column). But M_0 occurs very rarely anyway.
3) M_1 (when not linked with M_0) remains at 67%. For every three symbols o relating to M_1 (with no dot in the middle and unreduced by M_0) one success of the type ⊙ is added.
4) The role of extra successes (for M_1) in the value of a small-card system is analysed in all the summaries.
5) Thanks to this it is possible to correct the excessive advantage of M_1 over L and Q, which amounts to 17% in the tables (even though it should only be about 10½%). The summaries take this into account.

Test problems

To work out the efficiency of a small-card system it is necessary to construct statistical tables for many different defensive problems. In the next few chapters we shall test the workings of nearly all small-card systems in the following problems:

2 – 3 (4)   3 – 4 (4)   4 – 5 (5)
2 – 3 (5)   3 – 4 (5)   5 – 6 (6)

where the number in brackets refers to the number of hidden small cards (in partner's and declarer's hands).

PROBLEM 2-3(4)

In this problem there are three basic possibilities :

xx
xxx
Hxx

generating 16 possibilities in all.

The hidden hands contain the four lowest cards :

5 4 3 2

An illustration of Problem 2-3(4) is this defensive problem :

J976

1. xx
2. xxx
3. Qxx

AK108

1. Qxx
2. Qx
3. xx

2-3(4)		CLA			QQL		MQ	
⊗x	54	5	△		54	●		
	53	5	△		53	△		●
	52	5	△		52	△		●
	43	4	·	⊙△	43	●		
	42	4	·	⊙△	42	△		●
	32	3	○		32	●		
⊗xx	543	5	△		53	△		●
	542	5	△		52	△		●
	532	5	△		52	△		●
	432	4	+	△	42	△		●
Hx⊗	H54	4		○	45	●		
	H53	3	⊙		35	●		
	H52	2	●					
	H43	3	⊙		34	●		
	H42	2	●					
	H32	2	●					
75.35%	●		3	3		9	9	16
	○		3	6				
	□							
	△		6	7		7	7	
	⊕		15	16		16	16	16
	⊙		10	10		13	13	16

		MUD			QL		QLQ	
2-3(4)		BT			ML		MLM	
⊗x	54	5	●					
	53	5	●					
	52	5	●					
	43	4	+		43	●		
	42	4	+		42	●		
	32	3			32	●		
x⊗x	543	4	·		45	⊡		●
	542	4	·		45	⊡		●
	532	3	·		35	⊡		●
	432	3	·		34	⊡		●
Hx⊗	H54	4			45	□		●
	H53	3	+		35	□		●
	H52	2	●					
	H43	3	+		34	□		●
	H42	2	●					
	H32	2	●					
74.17%	●		6	6		9	9	16
	○							
	□					7	7	
	△							
	⊕		14	14		16	16	16
	⊙		10	10		13	13	16

2-3(4) REV

x⊗	54 53 52 43 42 32	4 3 + 2 ● 3 + 2 ● 2 ●	45 ● 35 ● 34 ●
x⊗x	543 542 532 432	4 · 4 · 3 · 3 ·	43 ⊡ ● 42 ⊡ ● 32 ⊡ ● 32 ⊡ ●
H⊗x	H54 H53 H52 H43 H42 H32	5 ● 5 ● 5 ● 4 + 4 + 3	43 □ ● 42 □ ● 32 □ ●
74.17 %	● ○ □ △	6 6	9 9 16 7 7
	⊕ ⊙	14 14 10 10	16 16 16 13 13 16

2-3(4) JOU LLQ LLM

⊗x	54 53 52 43 42 32	5 ● 5 ● 5 ● 4 ⊙ 4 ⊙ 3 ○	 43 ● 42 ● 32 ●
xx⊗	543 542 532 432	3 + □ 2 ⊡ 2 ⊡ 2 ⊡	35 □ ● 25 □ ● 25 ⊡ ● 24 ⊡
Hx⊗	H54 H53 H52 H43 H42 H32	4 ○ 3 · ⊡⊙ 2 □ 3 · ⊡⊙ 2 □ 2 □	45 ● 35 ⊡ ● 25 □ ● 34 ● 24 □ ● 23 ●
75.35 %	● ○ □ △	3 3 3 4 6 9	9 9 16 7 7
	⊕ ⊙	15 16 10 10	16 16 16 13 13 16

2-3(4) QQM

⊗x	54 53 52 43 42 32	5 △ 5 △ 5 △ 4 · ●△ 4 · ●△ 3 ○	54 △ ● 53 △ ● 52 ● 43 △ ● 42 ● 32 ●
⊗xx	543 542 532 432	5 △ 5 △ · 5 △ 4 + △	54 △ ● 54 △ ● 53 △ ● 43 △ ●
Hx⊗	H54 H53 H52 H43 H42 H32	4 ○ 3 ⊙ 2 ● 3 ⊙ 2 ● 2 ●	45 ● 35 ● 34 ●
75.35 %	● ○ □ △	3 3 3 6 6 7	9 9 16 7 7
	⊕ ⊙	15 16 10 10	16 16 16 13 13 16

2-3(4) LQ LQL

⊗x	54 53 52 43 42 32	5 ● 5 ● 5 ● 4 + 4 + 3	 43 △ ● 42 △ ● 32 △ ●
x⊗x	543 542 532 432	4 · 4 · 3 · 3 ·	43 △ ● 42 △ ● 32 △ ● 32 △ ●
Hx⊗	H54 H53 H52 H43 H42 H32	4 3 + 2 ● 3 + 2 ● 2 ●	45 ● 35 ● 34 ●
74.17 %	● ○ □ △	6 6	9 9 16 7 7
	⊕ ⊙	14 14 10 10	16 16 16 13 13 16

2–3(4) **LM LML**

x x	54	4	+		45	◐		
	53	3	+		35	◐		
	52	2	◐		25	◐		
	43	3	+		34	◐		
	42	2	◐		24	◐		
	32	2	◐		23	◐		
x x x	543	4	·		43	●		
	542	4	·		42	●		
	532	3	·		32	●		
	432	3	·		32	●		
H x x	H54	4			45			
	H53	3			35			
	H52	2	○		25			
	H43	3			34			
	H42	2	○		24			
	H32	2	○		23			
64.17 %	●					4	4	4
	○		6	6		12	12	12
	□							
	Δ							
	○		13	13		16	16	16
	○		8	8		12	12	12

2–3(4) **QM MML**

x x	54	4	+	◐	45	◐		
	53	3	◐		35	◐		
	52	2	◐		25	◐		
	43	3	◐		34	◐		
	42	2	◐		24	◐		
	32	2	◐		23	◐		
x x x	543	5	●					
	542	5	●					
	532	5	●					
	432	4	·	●	42	●		
H x x	H54	4		○	45	○		
	H53	3	○		35	○		
	H52	2	○		25	○		
	H43	3	○		34	○		
	H42	2	○		24	○		
	H32	2	○		23	○		
77.57 %	●		3	4		4	4	4
	○		10	12		12	12	12
	□							
	Δ							
	○		15	16		16	16	16
	○		11	12		12	12	12

2–3(4) **COMBINE**

x x	54	4	+	◐	45	◐		
	53	3	◐		35	◐		
	52	2	◐		25	◐		
	43	3	◐		34	◐		
	42	2	◐		24	◐		
	32	2	◐		23	◐		
x x x	543	5	●					
	542	5	●					
	532	5	●					
	432	4	·	●	43	●		
H x x	H54	4		○	45	○		
	H53	3	○		35	○		
	H52	2	○		25	○		
	H43	3	○		34	○		
	H42	2	○		24	○		
	H32	2	○		23	○		
77.57 %	●		3	4		4	4	4
	○		10	12		12	12	12
	□							
	Δ							
	○		15	16		16	16	16
	○		11	12		12	12	12

SUMMARY OF 2-3(4)

Using the indicators ⊕⊙ in Problem 2-3(4) there are four groups of systems:

Efficiency		I_0	I_1	II_0	II_1	II_2	Systems
		Indicators					
77.57 %	⊕	15	16	16	16	16	COMBINE QM MMQ MML
	⊙	11	12	12	12	12	
75.35 %	⊕	15	16	16	16	16	JOU CLA LLQ QQL LLM QQM MQ
	⊙	10	10	13	13	16	
74.17 %	⊕	14	14	16	16	16	REV MUD BT LQ ML QL QLQ LQL MLM
	⊙	10	10	13	13	16	
64.17 %	⊕	13	13	16	16	16	LM LML
	⊙	8	8	12	12	12	

The signal M_0 did not occur at all.

Additional successes of the type ⊙ for signal M_1 occured as follows:

I_0	I_1	II_0	II_1	II_2	
2	2	2	2	2	for COMBINE MM QM
1	1	2	2	2	for LM

This correspond to a 17 % advantage over L and Q which, as the author has admitted in the third edition, is somewhat excessive. After reducing this to the more reasonable value of 10.5 % (see „Evaluation of signals" and page 36) we get the following efficiences:

1) 75.35 % CLA JOU MQ QQ LL
2) 74.17 % REV MUD BT ML QL LQ
3) 73.75 % COMBINE MM QM
4) 61.62 % LM

N.B. In the summaries the third part of the name (S_s) has not been taken into account as two systems differing only in S_s have identical indicators.

PROBLEM 2-3(5)

In this problem there are three basic possibilities:

xx
xxx
Hxx

generating 30 possibilities in all.

The hidden hands contain the five lowest small cards:

6 5 4 3 2

Problem 2-3(5) can be illustrated as follows:

J109
1. xx
2. xxx
3. Qxx
AK87
1. Qxxx
2. Qxx
3. xxx

2-3(5)				CLA	QQL MQ	
⊗x	65	6	△		65	●
	64	6	△		64	△
	63	6	△		63	△
	62	6	△		62	△
	54	5	·		54	●
	53	5	·		53	△
	52	5	·		52	△
	43	4	·		43	●
	42	4	·		42	△
	32	3	○		32	●
⊗xx	654	6	◬		64	◬
	653	6	◬		63	◬
	652	6	◬		62	◬
	643	6	◬		63	◬
	642	6	◬		62	◬
	632	6	◬		62	◬
	543	5	+		53	◬
	542	5	+		52	◬
	532	5	+		52	◬
	432	4			42	◬
Hx⊗	H65	5			56	●
	H64	4	+		46	●
	H63	3	⊙		36	●
	H62	2	●			
	H54	4	+		45	●
	H53	3	⊙		35	●
	H52	2	●			
	H43	3	⊙		34	●
	H42	2	●			
	H32	2	●			
	●		4	4		14 14 14
	○		4	4		
72.44%	□					
	△		10	10		16 16 16
	⊕		28	28		30 30 30
	⊙		18	18		24 24 24

2-3(5)				MUD QLQ	BT ML QL MLM	
⊗x	65	6	●			
	64	6	●			
	63	6	●			
	62	6	●			
	54	5	·		54	●
	53	5	·		53	●
	52	5	·		52	●
	43	4	+		43	●
	42	4	+		42	●
	32	3			32	●
x⊗x	654	5	+		56	⊡
	653	5	+		56	⊡
	652	5	+		56	⊡
	643	4	·		46	⊡
	642	4	·		46	⊡
	632	3	·		36	⊡
	543	4	·		45	⊡
	542	4	·		45	⊡
	532	3	·		35	⊡
	432	3	·		34	⊡
Hx⊗	H65	5			56	□
	H64	4			46	□
	H63	3	+		36	□
	H62	2	●			
	H54	4			45	□
	H53	3	+		35	□
	H52	2	●			
	H43	3	+		34	□
	H42	2	●			
	H32	2	●			
	●		8	8		14 14 14
	○					
71.56%	□					16 16 16
	△					
	⊕		26	26		30 30 30
	⊙		18	18		24 24 24

2-3(5) REV

$\overset{\curvearrowleft}{\times\otimes}$	65	5		56	●
	64	4	+	46	●
	63	3	·	36	●
	62	2	●		
	54	4	+	45	●
	53	3	·	35	●
	52	2	●		
	43	3	·	34	●
	42	2	●		
	32	2	●		
$\times\overset{\curvearrowright}{\otimes\times}$	654	5	·	54	⊡
	653	5	·	53	⊡
	652	5	·	52	⊡
	643	4	·	43	⊡
	642	4	·	42	⊡
	632	3	+	32	⊡
	543	4	·	43	⊡
	542	4	·	42	⊡
	532	3	+	32	⊡
	432	3	+	32	⊡
$H\overset{\curvearrowright}{\otimes\times}$	H65	6	●		
	H64	6	●		
	H63	6	●		
	H62	6	●		
	H54	5	+	54	□
	H53	5	+	53	□
	H52	5	+	52	□
	H43	4		43	□
	H42	4		42	□
	H32	3		32	□
71.56 %	●	8 8		44 44 44	
	○				
	□			16 16 16	
	△				
	⊕	26 26		30 30 30	
	⊙	18 18		24 24 24	

2-3(5) JOU LLQ LLM

$\overset{\curvearrowright}{\otimes\times}$	65	6	●		
	64	6	●		
	63	6	●		
	62	6	●		
	54	5	⊙	54	●
	53	5	⊙	53	●
	52	5	⊙	52	●
	43	4	·	43	●
	42	4	·	42	●
	32	3		32	●
$\overset{\curvearrowleft}{\times\times\otimes}$	654	4		46	□
	653	3	+	36	⊡
	652	2	⊡	26	⊡
	643	3	+	36	⊡
	642	2	⊡	26	⊡
	632	2	⊡	26	⊡
	543	3	+	35	⊡
	542	2	⊡	25	⊡
	532	2	⊡	25	⊡
	432	2	⊡	24	⊡
$H\overset{\curvearrowleft}{\times\otimes}$	H65	5	○	56	●
	H64	4	+	46	⊡
	H63	3	·	36	□
	H62	2	□	26	□
	H54	4	+	45	●
	H53	3	·	35	□
	H52	2	□	25	□
	H43	3	·	34	●
	H42	2	□	24	□
	H32	2	□	23	⊙
72.44 %	●	4 4		44 44 44	
	○	4 4			
	□	10 10		16 16 16	
	△				
	⊕	28 28		30 30 30	
	⊙	18 18		24 24 24	

2-3(5) QQM MQM

$\overset{\curvearrowright}{\otimes\times}$	65	6	△	65	△
	64	6	△	64	△
	63	6	△	63	△
	62	6	△	62	●
	54	5	·	54	△
	53	5	·	53	△
	52	5	·	52	●
	43	4	·	43	△
	42	4	·	42	●
	32	3	○	32	●
$\overset{\curvearrowright}{\otimes\times}\times$	654	6	◬	65	◬
	653	6	◬	65	◬
	652	6	◬	65	◬
	643	6	◬	64	◬
	642	6	◬	64	◬
	632	6	◬	63	◬
	543	5	+	54	◬
	542	5	+	54	◬
	532	5	+	53	◬
	432	4		43	◬
$H\overset{\curvearrowleft}{\times\otimes}$	H65	5		56	●
	H64	4	+	46	●
	H63	3	⊙	36	●
	H62	2	●		
	H54	4	+	45	●
	H53	3	⊙	35	●
	H52	2	●		
	H43	3	⊙	34	●
	H42	2	●		
	H32	2	●		
72.44 %	●	4 4		44 44 44	
	○	4			
	□				
	△	10		16 16 16	
	⊕	28 28		30 30 30	
	⊙	18 18		24 24 24	

2-3(5) LQ LQL

$\overset{\curvearrowright}{\otimes\times}$	65	6	○		
	64	6	●		
	63	6	○		
	62	6	●		
	54	5	·	54	△
	53	5	·	53	△
	52	5	·	52	△
	43	4	+	43	△
	42	4	+	42	△
	32	3		32	△
$\times\overset{\curvearrowright}{\otimes\times}$	654	5	+	54	△
	653	5	+	53	△
	652	5	+	52	△
	643	4	·	43	◬
	642	4	·	42	◬
	632	3	+	32	◬
	543	4	·	43	◬
	542	4	·	42	◬
	532	3	+	32	◬
	432	3	+	32	◬
$H\overset{\curvearrowleft}{\times\otimes}$	H65	5		56	●
	H64	4		46	●
	H63	3	·	36	●
	H62	2	●		
	H54	4		45	●
	H53	3	·	35	●
	H52	2	●		
	H43	3	·	34	●
	H42	2	●		
	H32	2	●		
71.56 %	●	8 8		44 44 44	
	○				
	□				
	△			16 16 16	
	⊕	26 26		30 30 30	
	⊙	18 18		24 24 24	

2-3(5) LM LML

×⊗	65	5	+	56	⊙
	64	4	+	46	⊙
	63	3	·	36	⊙
	62	2	⊙	26	⊙
	54	4	+	45	⊙
	53	3	·	35	⊙
	52	2	⊙	25	⊙
	43	3	·	34	⊙
	42	2	⊙	24	⊙
	32	2	⊙	23	⊙
×⊗×	654	5	·	54	●
	653	5	·	53	●
	652	5	·	52	●
	643	4	·	43	●
	642	4	·	42	●
	632	3		32	●
	543	4	·	43	●
	542	4	·	42	●
	532	3		32	●
	432	3		32	●
H×⊗	H65	5		56	○
	H64	4		46	○
	H63	3	+	36	○
	H62	2	○	26	○
	H54	4		45	○
	H53	3	+	35	○
	H52	2	○	25	○
	H43	3	+	34	○
	H42	2	○	24	○
	H32	2	○	23	○

64.44 %	●			10	10	10
	○	8	8	20	20	20
	□					
	△					
	⊕	24	24	30	30	30
	⊙	15	15	23	23	23

2-3(5) QM MML

×⊗	65	5	+	56	⊙
	64	4	·	46	⊙
	63	3	⊙	36	⊙
	62	2	⊙	26	⊙
	54	4	·	45	⊙
	53	3	⊙	35	⊙
	52	2	⊙	25	⊙
	43	3	⊙	34	⊙
	42	2	⊙	24	⊙
	32	2	⊙	23	⊙
⊗××	654	6	●		
	653	6	●		
	652	6	●		
	643	6	●		
	642	6	●		
	632	6	●		
	543	5	·	53	●
	542	5	·	52	●
	532	5	·	52	●
	432	4		42	●
H×⊗	H65	5		56	○
	H64	4	+	46	○
	H63	3	○	36	○
	H62	2	○	26	○
	H54	4	+	45	○
	H53	3	○	35	○
	H52	2	○	25	○
	H43	3	○	34	○
	H42	2	○	24	○
	H32	2	○	23	○

75.11 %	●	6	6	10	10	10
	○	14	14	20	20	20
	□					
	△					
	⊕	28	28	30	30	30
	⊙	20	20	23	23	23

2-3(5) COMBINE QMQ MMQ

×⊗	65	5	+	56	⊙
	64	4	·	46	⊙
	63	3	⊙	36	⊙
	62	2	⊙	26	⊙
	54	4	·	45	⊙
	53	3	⊙	35	⊙
	52	2	⊙	25	⊙
	43	3	⊙	34	⊙
	42	2	⊙	24	⊙
	32	2	⊙	23	⊙
⊗××	654	6	●		
	653	6	●		
	652	6	●		
	643	6	●		
	642	6	●		
	632	6	●		
	543	5	·	54	●
	542	5	·	54	●
	532	5	·	53	●
	432	4		43	●
H×⊗	H65	5		56	○
	H64	4	+	46	○
	H63	3	○	36	○
	H62	2	○	26	○
	H54	4	+	45	○
	H53	3	○	35	○
	H52	2	○	25	○
	H43	3	○	34	○
	H42	2	○	24	○
	H32	2	○	23	○

75.11 %	●	6	6	10	10	10
	○	14	14	20	20	20
	□					
	△					
	⊕	28	28	30	30	30
	⊙	20	20	23	23	23

SUMMARY OF 2-3(5)

		Indicators					
Efficiency		I_0	I_1	II_0	II_1	II_2	Systems
75.11 %	⊕	28	28	30	30	30	COMBINE QM QMQ MML MMQ
	⊙	20	20	23	23	23	
72.44 %	⊕	28	28	30	30	30	CLA QQL QQM LLQ LLM JOU MQ MQM
	⊙	18	18	24	24	24	
71.56 %	⊕	26	26	30	30	30	MUD REV BT ML MLM QL QLQ LQ LQL
	⊙	18	18	24	24	24	
64.44 %	⊕	24	24	30	30	30	LM LML
	⊙	15	15	23	23	23	

Signal M_0 did not occur.

Additional successes of the type ⊙ for signal M_1 occured as follows:

I_0	I_1	II_0	II_1	II_2	
2	2	3	3	3	for COMBINE MM QM
1	1	3	3	3	for LM

This correspond to a 17 % advantage over L and Q which, as the author has admitted in the third edition, is somewhat excessive.

After reducing this to the more reasonable value of 10.5 % (see: „Evaluation of signals" and page 36) we get the following efficiences:

1) 72.73 % COMBINE MM QM
2) 72.44 % CLA JOU MQ QQ LL
3) 71.56 % REV MUD BT QL LQ ML
4) 62.74 % LM

N.B. In the summaries the third part of the name (S_5) has not been taken into account as two systems differing only in S_5 have identical indicators.

PROBLEM 3-4(4)

In this problem there are four basic possibilities:

xxx Hxx
xxxx Hxxx

generating 15 possibilities in all.

The hidden hands contain the four lowest small cards:

5 4 3 2

An illustration of Problem 3-4(4) is:

876

1. xxx
2. xxxx
3. Qxx
4. Qxxx

AJ109

1. KQx
2. KQ
3. Kxx
4. Kx

3-4(4) CLA

⊗xx	543	5 △ ○	53 ○
	542	5 △ ○	52 △ ○
	532	5 △ ○	52 △ ○
	432	4 ⊡ ○	42 ○
⊗xxx	5432	5 △ ○	52 △ ○
H×⊗	H54	4 □ ○	45 ○
	H53	3 △	35 ○
	H52	2 △	25 ○
	H43	3 △	34 △ ○
	H42	2 △	24 △ ○
	H32	2 △	23 △ ○
H××⊗	H543	3 △	34 △ ○
	H542	2 △	24 △ ○
	H532	2 △	23 △ ○
	H432	2 △	23 △ ○
76.59 %	○	6	5 8 15
	○		
	□	2	
	△	13 9	10 7
	¬		
	⊕	15 15	15 15 15
	⊙	9 11	11 12 15

3-4(4) MUD

×⊗×	543	4 ⊡	45 ⊡ ○
	542	4 ⊡	45 ⊡ ○
	532	3 ¬ □	35 □ ○
	432	3 ¬ □	34 ¬ □ ○
⊗xxx	5432	5 ○	
H×⊗	H54	4 □	45 □ ○
	H53	3 ¬ △⊡	35 ⊡ ○
	H52	2 △	25 ○
	H43	3 ¬ △⊡	34 ¬ △⊡ ○
	H42	2 △	24 △ ○
	H32	2 △	23 △ ○
H××⊗	H543	3 ¬ △	34 ¬ △ ○
	H542	2 △	24 △ ○
	H532	2 △	23 △ ○
	H432	2 △	23 △ ○
65.56 %	○	1 1	2 2 15
	○		
	□	3 5	5 6
	△	6 9	5 7
	¬	5	3
	⊕	14 15	14 15 15
	⊙	8 8	9 9 15

3-4 (4) REV

×⊗×	543 542 532 432	4 ㄱ· ⊡ 4 ㄱ· ⊡ 3 ㄱ· ⊡ 3 ㄱ· ⊡	43 ⊡ ⊙ 42 ⊡ ⊙ 32 ⊡ ⊙ 32 ⊡ ⊙
×⊗××	5432	4 ㄱ ⊙	45 ⊙
H⊗×	H54 H53 H52 H43 H42 H32	5 ⊙ 5 ⊙ 5 ⊙ 4 ㄱ· □ 4 ㄱ· □ 3 ㄱ· □	 43 □ ⊙ 42 □ ⊙ 32 □ ⊙
H××⊗	H543 H542 H532 H432	3 ㄱ ⊙ 2 ⊙ 2 ⊙ 2 ⊙	34 ⊙
79.70 %	⊙ ○ □ △ ㄱ	6 8 7 9	8 8 15 7 7
	⊕ ⊚	13 15 10 12	15 15 15 12 12 15

3-4 (4) BT QL

×⊗×	543 542 532 432	4 ㄱ· 4 ㄱ· 3 ㄱ· 3 ㄱ·	45 ⊡ ⊙ 45 ⊡ ⊙ 35 ⊡ ⊙ 34 ⊡ ⊙
⊗×××	5432	5 ⊙	
H×⊗	H54 H53 H52 H43 H42 H32	4 ㄱ 3 ㄱ 2 ⊙ 3 ㄱ 2 ⊙ 2 ⊙	45 □ ⊙ 35 □ ⊙ 34 □ ⊙
H×⊗×	H543 H542 H532 H432	4 ㄱ· 4 ㄱ· 3 ㄱ· 3 ㄱ·	43 ⊙ 42 ⊙ 32 ⊙ 32 ⊙
68.00 %	⊙ ○ □ △ ㄱ	4 4 11 11	8 8 15 7 7
	⊕ ⊚	12 12 8 8	15 15 15 12 12 15

3-4 (4) JOU

××⊗	543 542 532 432	3 □ 2 ⊡ 2 ⊡ 2 ⊡	35 □ ○ 25 ⊡ ○ 25 ⊡ ○ 24 ⊡ ○
××⊗×	5432	3 ⊙	32 □ ⊙
H×⊗	H54 H53 H52 H43 H42 H32	4 △ 3 · ⊡△ 2 □ 3 · ⊡△ 2 □ 2 □	45 ⊙ ○ 35 ⊡ ○ 25 □ ○ 34 ⊙ 24 □ ○ 23 ⊙
H×⊗×	H543 H542 H532 H432	4 △ 4 △ 3 + △ 3 + △	43 ⊙ 42 ⊙ 32 ⊡ ⊙ 32 ⊡ ○
66.96 %	⊙ ○ □ △ ㄱ	1 6 9 3 5	5 8 15 10 7
	⊕ ⊚	13 15 7 8	15 15 15 11 12 15

3-4 (4) QQL

⊗××	543 542 532 432	5 △ ○ 5 △ ○ 5 △ ○ 4 □ ⊙	53 ⊙ 52 ○ 52 ○ 42 ○
⊗×××	5432	5 △ ○	54 ⊙
H×⊗	H54 H53 H52 H43 H42 H32	4 ⊡ ⊙ 3 △ 2 △ 3 △ 2 △ 2 △	45 ○ 35 △ ⊙ 25 △ ⊙ 34 ⊙ 24 △ ⊙ 23 ⊙
H××⊗	H543 H542 H532 H432	3 △ 2 △ 2 △ 2 △	35 △ ⊙ 25 △ ○ 25 △ ○ 24 △ ○
77.19 %	⊙ ○ □ △ ㄱ	6 2 13 9	8 8 15 7 7
	⊕ ⊚	15 15 9 11	15 15 15 12 12 15

3-4(4) LLQ

××⊗	543 542 532 432	3 ⊡ 2 ⊡ 2 ⊡ 2 ⊡	35 □ ◉ 25 ⊡ ◉ 25 ⊡ ◉ 24 ⊡ ◉
⊗×××	5432	5 □ ◉	54 ◉
H×⊗	H54 H53 H52 H43 H42 H32	4 △ ◉ 3 ⊡ 2 □ 3 ⊡ 2 □ 2 □	45 ◉ 35 ⊡ ◉ 25 □ ◉ 34 ◉ 24 □ ◉ 23 ○
H⊗××	H543 H542 H532 H432	5 ⊡ ◉ 5 ⊡ ◉ 5 ⊡ ◉ 4 △ ◉	53 ◉ 52 ◉ 52 ◉ 42 ◉
77.19 %	◉	6	8 8 15
	○		
	□	13 9	7 7
	△	2	
	¬		
	⦶	15 15	15 15 15
	⊙	9 11	12 12 15

3-4(4) QLQ

×⊗×	543 542 532 432	4 ㅋ 4 ㅋ 3 ㅋ 3 ㅋ	45 ⊡ ◉ 45 ⊡ ◉ 35 ⊡ ◉ 34 ⊡ ◉
⊗×××	5432	5 ◉	
H×⊗	H54 H53 H52 H43 H42 H32	4 ¬ 3 ¬ 2 ◉ 3 ¬ 2 ◉ 2 ◉	45 □ ◉ 35 □ ◉ 34 □ ◉
H×⊗×	H543 H542 H532 H432	4 ㅋ 4 ㅋ 3 ㅋ 3 ㅋ	43 ◉ 42 ◉ 32 ◉ 32 ◉
68.00 %	◉	4 4	8 8 15
	○		
	□		7 7
	△		
	¬	11 11	
	⦶	12 12	15 15 15
	⊙	8 8	12 12 15

3-4(4) QM

⊗××	543 542 532 432	5 ◉ 5 ◉ 5 ◉ 4 ◉	 42 ⊙ ◉
×⊗××	5432	4 + ◉	45 ⊙ ◉
H×⊗	H54 H53 H52 H43 H42 H32	4 △ 3 △ 2 ◉ 3 △ 2 ◉ 2 ◉	45 ○ ◉ 35 ◉ 34 ◉
H×⊗×	H543 H542 H532 H432	4 · △ 4 · △ 3 △ 3 △	43 ◉ 42 ○ ◉ 32 ◉ 32 ◉
82.96 %	◉	6 8	11 15 15
	○		4
	□		
	△	4 7	
	¬		
	⦶	13 15	15 15 15
	⊙	10 12	13 15 15

3-4(4) MML

⊗××	543 542 532 432	5 ⊙ 5 ⊙ 5 ⊙ 4 ¬ ⊙	53 ⊙ ◉ 52 ◉ 52 ◉ 42 ◉
×××⊗	5432	2 ○ ◉	25 ○ ◉
H×⊗	H54 H53 H52 H43 H42 H32	4 ㅋ ◉ 3 ◉ 2 ⊙ ◉ 3 ◉ 2 ⊙ ◉ 2 ⊙ ◉	45 ◉ 25 ⊙ ◉ 24 ◉ 23 ◉
H⊗××	H543 H542 H532 H432	5 ○ 5 ○ 5 ○ 4 ㅋ ○	54 ◉ 54 ◉ 53 ○ ◉ 43 ◉
88.48 %	◉	2 7	12 15 15
	○	10 8	3
	□		
	△		
	¬	3	
	⦶	14 15	15 15 15
	⊙	10 12	13 15 15

3-4 (4) LM

×⊗×	543 542 532 432	4 · ⊡ 4 · ⊡ 3 ⊡ 3 ⊡	43 ● 42 ⊙ ● 32 ● 32 ●
×⊗××	5432	4 ●	45 ⊙ ●
H×⊗	H54 H53 H52 H43 H42 H32	4 □ 3 □ 2 ● 3 □ 2 ● 2 ●	45 ○ ● 35 ● 34 ●
H⊗××	H543 H542 H532 H432	5 ● 5 ● 5 ● 4 + ●	 42 ○ ●
82.96%	● ○ □ △ ┐	6 8 4 7	11 15 15 4
	⊕ ⊙	13 15 10 12	15 15 15 13 15 15

3-4 (4) MQ

⊗××	543 542 532 432	5 ● 5 ● 5 ● 4 ┐ ●	 42 ●
××⊗×	5432	3 ┐ ●	32 ●
H×⊗	H54 H53 H52 H43 H42 H32	4 ┐ △ 3 ┐ △ 2 ● 3 ┐ △ 2 ● 2 ●	45 △ ● 35 △ ● 34 △ ●
H×⊗×	H543 H542 H532 H432	4 ┐ △ 4 ┐ △ 3 ┐ △ 3 ┐ △	45 △ ● 45 △ ● 35 △ ● 34 △ ●
79.70%	● ○ □ △ ┐	6 8 7 9	8 8 15 7 7
	⊕ ⊙	13 15 10 12	15 15 15 12 12 15

3-4 (4) ML

×⊗×	543 542 532 432	4 ┐ ⊡ 4 ┐ ⊡ 3 ┐ ⊡ 3 ┐ ⊡	45 ⊡ ● 45 ⊡ ● 35 ⊡ ● 34 ⊡ ●
××⊗×	5432	3 ┐ ●	32 ●
H×⊗	H54 H53 H52 H43 H42 H32	4 ┐ □ 3 ┐ □ 2 ● 3 ┐ □ 2 ● 2 ●	45 □ ● 35 □ ● 34 □ ●
H⊗××	H543 H542 H532 H432	5 ● 5 ● 5 ● 4 ┐ ●	 42 ●
79.70%	● ○ □ △ ┐	6 8 7 9	8 8 15 7 7
	⊕ ⊙	13 15 10 12	15 15 15 12 12 15

3-4 (4) COMBINE

⊗××	543 542 532 432	5 △ ● 5 △ ● 5 △ ● 4 ┐ ●	54 ● 54 ● 53 ● 43 ⊙ ●
⊗×××	5432	5 △ ●	52 ●
H×⊗	H54 H53 H52 H43 H42 H32	4 ┐ △ 3 △ 2 ● 3 △ 2 ● 2 ●	45 ● 35 ● 34 ●
H×⊗×	H543 H542 H532 H432	4 ┐ △ 4 ┐ △ 3 △ 3 △	43 ○ ● 42 ● 32 ● 32 ●
84.07%	● ○ □ △ ┐	3 8 8 7 4	13 15 15 2
	⊕ ⊙	14 15 10 12	15 15 15 14 15 15

SUMMARY OF 3-4 (4)

		Indicators					
Efficiency		I_0	I_1	II_0	II_1	II_2	Systems
84.07 %	⊕	14	15	15	15	15	COMBINE
	⊙	10	12	14	15	15	
83.48 %	⊕	14	15	15	15	15	MMQ MML
	⊙	10	12	13	15	15	
82.96 %	⊕	13	15	15	15	15	QM QMQ LM LML
	⊙	10	12	13	15	15	
79.70 %	⊕	13	15	15	15	15	REV MQ MQM ML MLM
	⊙	10	12	12	12	15	
77.19 %	⊕	15	15	15	15	15	QQL QQM LLQ LLM
	⊙	9	11	12	12	15	
76.59 %	⊕	15	15	15	15	15	CLA
	⊙	9	11	11	12	15	
68.00 %	⊕	12	12	15	15	15	BT QL QLQ LQ LQL
	⊙	8	8	12	12	15	
66.96 %	⊕	13	15	15	15	15	JOU
	⊙	7	8	11	12	15	
66.56 %	⊕	14	15	14	15	15	MUD
	⊙	8	8	9	9	15	

In Problem 3-4(4) the signal M_0 only appears a few times, and then is always linked with signal M_1, or has no negative effect.

Additional successes of the type ⊙ for signal M_1 occured only in the system MM and comprised:

1 in I_0 and 1 in I_1.

After reducing the advantage of M_1 from 17% to 10.5% (see summary of problems 2-3) the efficiency of system MM is reduced to 82.12%.

The final order is:

1) 84.07 % COMBINE
2) 82.96 % QM LM
3) 82.42 % MM
4) 79.70 % REV MQ ML
5) 77.19 % QQ LL
6) 76.59 % CLA
7) 68.00 % BT QL LQ
8) 66.96 % JOU
9) 65.56 % MUD

FOR THOSE WHO HAVE NO CONFIDENCE IN STATISTICS

The efficiency of small-card systems, expressed as a percentage, is based on several parameters (see page 34 and 35) whose values have been assigned largely intuitively (without exact documentation).

However, the order of systems in a given problem can generally be established without using percentages, but relying solely on the indicators ⊕⊙.

For example, look at the tables of successes for systems COMBINE and REV:

COMBINE	⊕	14	15	15	15	15
	⊙	10	12	14	15	15

REV	⊕	13	15	15	15	15
	⊙	10	12	12	12	15

Since every indicator for COMBINE is ⩾ the corresponding indicator for REV (and there are, in this case, no extra successes for the greater value of signal M_1), COMBINE is better than REV, irrespective of the value of the parameters:

Similarly it can be shown that:

COMBINE > QM = LM > REV

QQ > CLA > MUD ... etc.

The order of systems thus established is only marginally different from the order using percentages, as was the case in the first edition.

PROBLEM 3-4(5)

In this problem there are four basic possibilities:

xxx Hxx
xxxx Hxxx

generating 35 possibilities in all.

The hidden hands contain the five lowest small cards: 6 5 4 3 2

An illustration of Problem 3-4(5) is this defensive situation:

987

1. xxx
2. xxxx
3. Qxx
4. Qxxx

AJ10

1. KQxx
2. KQx
3. Kxxx
4. Kxx

3-4(5) CLA

⊗xx	654	6	Δ		64	∘		
	653	6	Δ		63	Δ		∘
	652	6	Δ		62	Δ		∘
	643	6	Δ		63	Δ		∘
	642	6	Δ		62	Δ		∘
	632	6	Δ		62	Δ		∘
	543	5	╕	⊡Δ	53	∘		
	542	5	╕	⊡Δ	52	Δ		∘
	532	5	╕	⊡Δ	52	Δ		∘
	432	4	┐	□	42	∘		
⊗xxx	6543	6	Δ		63	Δ		∘
	6542	6	Δ		62	Δ		∘
	6532	6	Δ		62	Δ		∘
	6432	6	Δ		62	Δ		∘
	5432	5	╕	Δ	52	Δ		∘
Hx⊗	H65	5	┐	□	56	∘		
	H64	4	╕	Δ⊡	46	∘		
	H63	3	Δ		36	∘		
	H62	2	Δ		26	∘		
	H54	4	╕	Δ⊡	45	Δ		
	H53	3	Δ		35	Δ		
	H52	2	Δ		25	Δ		
	H43	3	Δ		34	Δ		
	H42	2	Δ		24	Δ		
	H32	2	Δ		23	Δ		
Hxx⊗	H654	4	╕	Δ	45	Δ		
	H653	3	Δ		35	Δ		
	H652	2	Δ		25	Δ		
	H643	3	Δ		34	Δ		
	H642	2	Δ		24	Δ		
	H632	2	Δ		23	Δ		
	H543	3	Δ		34	Δ		
	H542	2	Δ		24	Δ		
	H532	2	Δ		23	Δ		
	H432	2	Δ		23	Δ		
68.95 %	∘					7	7	49
	○							
	□			5				
	Δ		26	30		28	28	46
	┐		9					
	∘		33	35		35	35	35
	⊚		29	30		24	24	23

3-4(5) MUD

x⊗x	654	5	╕	⊡	56	⊡		
	653	5	╕	⊡	56	⊡		
	652	5	╕	⊡	56	⊡		
	643	4	╕		46	⊡		
	642	4	╕		46	⊡		
	632	3	┐		36	□		
	543	4	╕		45	╕		⊡
	542	4	╕		45	╕		⊡
	532	3	┐		35	┐		⊡
	432	3	┐		34	┐		⊡
⊗xxx	6543	6	○					
	6542	6	○					
	6532	6	○					
	6432	6	○					
	5432	5	╕	○	52	○		
Hx⊗	H65	5	┐	□	56	□		
	H64	4	╕		46	□		
	H63	3	╕		36	⊡		
	H62	2	Δ		26	∘		
	H54	4	╕		45	╕		Δ□
	H53	3	╕		35	╕		Δ□
	H52	2	Δ		25	Δ		
	H43	3	╕		34	╕		Δ□
	H42	2	Δ		24	Δ		
	H32	2	Δ		23	Δ		
Hxx⊗	H654	4	┐		45	┐		Δ
	H653	3	╕		35	╕		Δ
	H652	2	Δ		25	Δ		
	H643	3	╕		34	╕		
	H642	2	Δ		24	Δ		
	H632	2	Δ		23	Δ		
	H543	3	╕		34	╕		Δ
	H542	2	Δ		24	Δ		
	H532	2	Δ		23	Δ		
	H432	2	Δ		23	Δ		
66.79 %	∘		4	5		6	6	6
	○							
	□			4		9	9	44
	Δ		40	40		9	9	45
	┐		21	16		41	41	
	∘		30	34		32	32	35
	⊚		26	34		23	23	26

3-4(5) REV

×⊗×	654	5	╕		54	⊡	
	653	5	╕		53	⊡	
	652	5	╕		52	⊡	
	643	4	·	╕	43	⊡	
	642	4	·	╕	42	⊡	
	632	3	╕		32	⊡	
	543	4	·	╕	43	⊡	
	542	4	·	╕	42	⊡	
	532	3	╕		32	⊡	
	432	3	╕		32	⊡	
×⊗××	6543	5	╕		56	●	
	6542	5	╕		56	●	
	6532	5	╕		56	●	
	6432	4	+	┐	46	●	
	5432	4	+	┐	45	⊡	●
H⊗×	H65	6	●				
	H64	6	●				
	H63	6	●				
	H62	6	●				
	H54	5	┐		54	□	
	H53	5	┐		53	□	
	H52	5	┐		52	□	
	H43	4		╕	43	□	
	H42	4		╕	42	□	
	H32	3	┐		32	□	
H××⊗	H654	4		┐	45	□	●
	H653	3	╕		35	●	
	H652	2	●				
	H643	3	╕		34	●	
	H642	2	●				
	H632	2	●				
	H543	3	╕		34	●	
	H542	2	●				
	H532	2	●				
	H432	2	●				

69.65 %	●	40	40	47	49	49
	○					
	□			48	46	46
	△					
	┐	46	25			
	○ ⊙	28 29	28 29	35 28	35 29	35 29

3-4(5) BT QL

×⊗×	654	5	·	╕	56	⊡	
	653	5	·	╕	56	⊡	
	652	5	·	╕	56	⊡	
	643	4	╕		46	⊡	
	642	4	╕		46	⊡	
	632	3	╕		36	⊡	
	543	4	╕		45	⊡	
	542	4	╕		45	⊡	
	532	3	╕		35	⊡	
	432	3	╕		34	⊡	
⊗×××	6543	6	●				
	6542	6	●				
	6532	6	●				
	6432	6	●				
	5432	5		●	52	□	●
H×⊗	H65	5		┐	56	□	
	H64	4	┐		46	□	
	H63	3	┐		36	□	
	H62	2	●				
	H54	4	┐		45	□	
	H53	3	┐		35	□	
	H52	2	●				
	H43	3	┐		34	□	
	H42	2	●				
	H32	2	●				
H×⊗×	H654	5	+	╕	54	●	
	H653	5	+	╕	53	●	
	H652	5	+	╕	52	⊡	●
	H643	4	╕		43	○	
	H642	4	╕		42	○	
	H632	3	╕		32	○	
	H543	4	╕		43	○	
	H542	4	╕		42	○	
	H532	3	╕		32	○	
	H432	3	╕		32	○	

67.40 %	●	8	9	47	49	49
	○					
	□			48	46	46
	△					
	┐	49	26			
	○ ⊙	28 46	29 29	35 23	35 22	35 23

3-4(5) JOU

××⊗	654	4			46	□	
	653	3	+		36	⊡	
	652	2	⊡		26	⊡	
	643	3	+		36	⊡	
	642	2	⊡		26	⊡	
	632	2	⊡		26	⊡	
	543	3	+		35	⊡	
	542	2	⊡		25	⊡	
	532	2	⊡		25	⊡	
	432	2	⊡		24	⊡	
××⊗×	6543	4	+		43	□	●
	6542	4	+		42	□	●
	6532	3	·		32	⊡	●
	6432	3	·		32	⊡	●
	5432	3	·		32	⊡	●
H×⊗	H65	5	△		56	●	
	H64	4			46	⊡	
	H63	3			36	□	
	H62	2	□		26	□	
	H54	4			45	●	
	H53	3			35	□	
	H52	2	□		25	□	
	H43	3			34	●	
	H42	2	□		24	□	
	H32	2	□		23	●	
H×⊗×	H654	5	△		54	●	
	H653	5	△		53	●	
	H652	5	△		52	●	
	H643	4	·		43	⊡	●
	H642	4	·		42	⊡	●
	H632	3			32	□	●
	H543	4	·		43	⊡	●
	H542	4	·		42	⊡	●
	H532	3			32	□	●
	H432	3			32	□	●

59.87 %	●			7	7	49
	○					
	□	40	40	28	28	46
	△	4	4			
	┐					
	○ ⊙	26 26	26 26	35 24	35 24	35 23

3-4(5) QQL

⊗××	654	6	△		64	△	●
	653	6	△		63	●	
	652	6	△		62	●	
	643	6	△		63	●	
	642	6	△		62	●	
	632	6	△		62	●	
	543	5	╕	□△	53	●	
	542	5	╕	□△	52	●	
	532	5	╕	□△	52	●	
	432	4	┐	□	42	●	
⊗×××	6543	6	△		65	●	
	6542	6	△		65	●	
	6532	6	△		65	●	
	6432	6	△		64	△	●
	5432	5	╕	△	54	●	
H×⊗	H65	5	┐	□	56	●	
	H64	4	╕	△□	46	△	
	H63	3	△		36	△	
	H62	2	△		26	△	
	H54	4	╕	△□	45	●	
	H53	3	△		35	△	
	H52	2	△		25	△	
	H43	3	△		34	●	
	H42	2	△		24	△	
	H32	2	△		23	●	
H××⊗	H654	4	╕	△	46	△	
	H653	3	△		36	△	
	H652	2	△		26	△	
	H643	3	△		36	△	
	H642	2	△		26	△	
	H632	2	△		26	△	
	H543	3	△		35	△	
	H542	2	△		25	△	
	H532	2	△		25	△	
	H432	2	△		24	△	

71.87 %	●			47	49	49
	○					
	□		5			
	△	26	30	48	46	46
	┐	9				
	○ ⊙	33 26	35 20	35 28	35 23	35 28

3-4(5) LLQ

××⊗	654	4	┐·	□	46	□	
	653	3	□		36	⊡	
	652	2	⊡		26	⊡	
	643	3	□		36	⊡	
	642	2	⊡		26	⊡	
	632	2	⊡		26	□	
	543	3	□		35	□	
	542	2	⊡		25	⊡	
	532	2	⊡		25	⊡	
	432	2	⊡		24	⊡	
⊗×××	6543	6	□		65	∘	
	6542	6	□		65	∘	
	6532	6	□		65	∘	
	6432	6	□		64	□	∘
	5432	5	┐·	□	54	∘	
H×⊗	H65	5	┐	Δ	56	∘	
	H64	4	┐·	⊡Δ	46	⊡	
	H63	3	⊡		36	□	
	H62	2	□		26	□	
	H54	4	┐·	⊡Δ	45	∘	
	H53	3	⊡		35	⊡	
	H52	2	□		25	□	
	H43	3	⊡		34	∘	
	H42	2	□		24	□	
	H32	2	□		23	∘	
H⊗××	H654	6	⊡		64	⊡	∘
	H653	6	⊡		63	∘	
	H652	6	⊡		62	∘	
	H643	6	⊡		63	∘	
	H642	6	⊡		62	∘	
	H632	6	⊡		62.	∘	
	H543	5	┐·	Δ⊡	53	∘	
	H542	5	┐·	Δ⊡	52	∘	
	H532	5	┐·	Δ⊡	52	∘	
	H432	4	┐	Δ	42	∘	
71.87 %	∘				47 49 45		
	○						
	□	26 30			48 46 46		
	Δ	5					
	┐	9					
	∘	34 35			[illegible]		
	⊙	20 20			[illegible]		

3-4(5) MML

⊗××	654	6	⊙		64	○	●
	653	6	⊙		63	⊙	●
	652	6	⊙		62	∘	
	643	6	⊙		63	⊙	●
	642	6	⊙		62	∘	
	632	6	⊙		62	∘	
	543	5	┐·		53	○	●
	542	5	┐·		52	∘	
	532	5	┐·		52	∘	
	432	4	┐·		42	∘	
×××⊗	6543	3	○		36	○	●
	6542	2	⊙		26	⊙	●
	6532	2	⊙		26	⊙	●
	6432	2	⊙		26	⊙	●
	5432	2	⊙		25	⊙	●
H×⊗	H65	5	┐		56	∘	
	H64	4	┐·		46	∘	
	H63	3	⊙		36	⊙	●
	H62	2	○		26	○	●
	H54	4	┐·		45	∘	
	H53	3	⊙		35	∘	
	H52	2	○		25	○	●
	H43	3	⊙		34	∘	
	H42	2	○		24	∘	
	H32	2	○		23	∘	
H⊗××	H654	6	○		65	∘	
	H653	6	○		65	∘	
	H652	6	○		65	∘	
	H643	6	○		64	⊙	●
	H642	6	○		64	⊙	●
	H632	6	○		63	○	⊙
	H543	5	┐·		54	∘	
	H542	5	┐·		54	∘	
	H532	5	┐·		53	⊙	●
	H432	4	┐·		43	∘	
71.05 %	∘				49 49 35		
	○	24 24			46 46		
	□						
	Δ						
	┐	11 11					
	∘	33 33			35 35 35		
	⊙	43 45			23 23 35		

3-4(5) GM

⊗××	654	6	∘				
	653	6	∘				
	652	6	∘				
	643	6	∘				
	642	6	∘				
	632	6	∘				
	543	5	+		53	⊙	∘
	542	5	+		52	⊙	∘
	532	5	+		52	⊙	∘
	432	4			42	○	∘
×⊗××	6543	5	·		56	⊙	∘
	6542	5	·		56	⊙	∘
	6532	5	·		56	⊙	∘
	6432	4	+		46	⊙	∘
	5432	4	+		45	⊙	∘
H×⊗	H65	5			56	○	∘
	H64	4			46	○	∘
	H63	3	Δ		36	∘	
	H62	2	∘				
	H54	4			45	○	∘
	H53	3	Δ		35	∘	
	H52	2	∘				
	H43	3	Δ		34	∘	
	H42	2	∘				
	H32	2	∘				
H×⊗×	H654	5			54	∘	
	H653	5			53	○	∘
	H652	5			52	○	∘
	H643	4	·		43	∘	
	H642	4	·		42	⊙	∘
	H632	3	Δ		32	∘	
	H543	4	·		43	∘	
	H542	4	·		42	⊙	∘
	H532	3	Δ		32	∘	
	H432	3	Δ		32	∘	
70.67 %	∘	40 40			43 49 35		
	○				46 46		
	□						
	Δ	6 6					
	┐						
	∘	25 25			[illegible]		
	⊙	20 23			[illegible]		

3-4(5) LQ

×⊗×	654	5	·	┐·	54	∘	
	653	5	·	┐·	53	∘	
	652	5	·	┐·	52	Δ	●
	643	4	┐·		43	∘	
	642	4	┐·		42	∘	
	632	3	┐·		32	∘	
	543	4	┐·		43	∘	
	542	4	┐·		42	∘	
	532	3	┐·		32	∘	
	432	3	┐·		32	∘	
⊗×××	6543	6	∘				
	6542	6	∘				
	6532	6	∘				
	6432	6	∘				
	5432	5		∘	52	Δ	∘
H×⊗	H65	5		┐	56	Δ	
	H64	4	┐		46	Δ	
	H63	3	┐·		36	Δ	
	H62	2	∘				
	H54	4	┐		45	Δ	
	H53	3	┐·		35	Δ	
	H52	2	∘				
	H43	3	┐·		34	Δ	
	H42	2	∘				
	H32	2	∘				
H×⊗×	H654	5	+	┐·	56	Δ	
	H653	5	+	┐·	56	Δ	
	H652	5	+	┐·	56	Δ	
	H643	4	┐·		46	Δ	
	H642	4	┐·		46	Δ	
	H632	3	┐		36	Δ	
	H543	4	┐·		45	Δ	
	H542	4	┐·		45	Δ	
	H532	3	┐		35	Δ	
	H432	3	┐		34	Δ	
67.40 %	∘	8 9			47 49 49		
	○						
	□						
	Δ				48 46 46		
	┐	49 26					
	∘	28 29			36 36 35		
	⊙	49 49			22 29 [illegible]		

3-4(5) LM

×⊗×	654	5	·		54	●		
	653	5	·		53	⊙		●
	652	5	·		52	○		●
	643	4	·		43	●		
	642	4	·		42	⊙		●
	632	3	□		32	●		
	543	4	·		43	●		
	542	4	·		42	○		
	532	3	□		32	●		
	432	3	⊡		32	●		
×⊗××	6543	5	·		56	⊙		●
	6542	5	·		56	○		●
	6532	5	·		56	⊙		●
	6432	4	+		46	⊙		●
	5432	4	+		45	○		●
H×⊗	H65	5			56	○		●
	H64	4			46	○		●
	H63	3	⊡		36	●		
	H62	2	●					
	H54	4			45	○		●
	H53	3	⊡		35	●		
	H52	2	●					
	H43	3	⊡		34	●		
	H42	2	●					
	H32	2	●					
H⊗××	H654	6	●					
	H653	6	●					
	H652	6	●					
	H643	6	●					
	H642	6	●					
	H632	6	●					
	H543	5			53	○		○
	H542	5			52	⊙		○
	H532	5			52	⊙		●
	H432	4			42	○		○
70.67%	●		40	40		43	43	35
	○					46	46	
	□		6	6				
	△							
	ㄱ							
	●		26	25		[illegible]	[illegible]	[illegible]
	⊙		19	20		[illegible]	[illegible]	[illegible]

3-4(5) MQ

⊗××	654	6	●					
	653	6	●					
	652	6	●					
	643	6	●					
	642	6	●					
	632	6	●					
	543	5	ㅋ		53	●		
	542	5	ㅋ		52	●		
	532	5	ㅋ		52	●		
	432	4		ㄱ	42	△	●	
××⊗×	6543	4		ㄱ	43	●		
	6542	4		ㄱ	42	△	●	
	6532	3	ㅋ		32	●		
	6432	3	ㅋ		32	●		
	5432	3	ㅋ		32	●		
H×⊗	H65	5	ㄱ		56	△		
	H64	4	+	ㅋ	46	△		
	H63	3	ㅋ		36	△		
	H62	2	●					
	H54	4	+	ㅋ	45	△		
	H53	3	ㅋ		35	△		
	H52	2	●					
	H43	3	ㅋ		34	△		
	H42	2	●					
	H32	2	●					
H×⊗×	H654	5	ㅋ		56	△		
	H653	5	ㅋ		56	△		
	H652	5	ㅋ		56	△		
	H643	4	·	ㅋ	46	△		
	H642	4	·	ㅋ	46	△		
	H632	3	ㄱ		36	△		
	H543	4	·	ㅋ	45	△		
	H542	4	·	ㅋ	45	△		
	H532	3	ㄱ		35	△		
	H432	3	ㄱ		34	△		
69.65%	●		40	40		47	43	43
	○							
	□							
	△					43	46	46
	ㄱ		46	25				
	●		26	23		[illegible]	[illegible]	35
	⊙		20	20		[illegible]	21	21

3-4(5) ML

×⊗×	654	5	ㅋ		56	⊡		
	653	5	ㅋ		56	⊡		
	652	5	ㅋ		56	⊡		
	643	4	·	ㅋ	46	⊡		
	642	4	·	ㅋ	46	⊡		
	632	3	ㄱ		36	⊡		
	543	4	·	ㅋ	45	⊡		
	542	4	·	ㅋ	45	⊡		
	532	3	ㄱ		35	⊡		
	432	3	ㄱ		34	⊡		
××⊗×	6543	4	+	ㅋ	43	●		
	6542	4	+	ㅋ	42	⊡	●	
	6532	3	ㅋ		32	●		
	6432	3	ㅋ		32	●		
	5432	3	ㅋ		32	●		
H×⊗	H65	5	ㄱ		56	□		
	H64	4		ㄱ	46	□		
	H63	3	ㅋ		36	□		
	H62	2	●					
	H54	4		ㄱ	45	□		
	H53	3	ㅋ		35	□		
	H52	2	●					
	H43	3	ㅋ		34	□		
	H42	2	●					
	H32	2	●					
H⊗××	H654	6	●					
	H653	6	●					
	H652	6	●					
	H643	6	●					
	H642	6	●					
	H632	6	●					
	H543	5	ㅋ		53	●		
	H542	5	ㅋ		52	●		
	H532	5	ㅋ		52	●		
	H432	4		ㄱ	42	□	●	
69.65%	●		40	40		47	43	49
	○							
	□					48	46	46
	△							
	ㄱ		46	25				
	●		32	28		35	35	35
	⊙		20	23		28	29	23

3-4(5) COMBINE

⊗××	654	6	△		65	●		
	653	6	△		65	●		
	652	6	△		65	●		
	643	6	△		64	●		
	642	6	△		64	●		
	632	6	△		63	△	●	
	543	5	·	ㅋ	54	⊙		●
	542	5	·	ㅋ	54	⊙		●
	532	5	·	ㅋ	53	○		●
	432	4	ㄱ		43	○		●
⊗×××	6543	6	△		63	△	●	
	6542	6	△		62	●		
	6532	6	△		62	●		
	6432	6	△		62	●		
	5432	5		△	52	□	●	
H×⊗	H65	5		ㄱ	56	●		
	H64	4	ㅋ		46	●		
	H63	3	△		36	●		
	H62	2	●					
	H54	4	ㅋ		45	●		
	H53	3	△		35	●		
	H52	2	●					
	H43	3	△		34	●		
	H42	2	●					
	H32	2	●					
H×⊗×	H654	5	+	ㅋ	54	○		●
	H653	5	+	ㅋ	53	○		●
	H652	5	+	ㅋ	52	⊡	●	
	H643	4	ㅋ		43	⊙		●
	H642	4	ㅋ		42	●		
	H632	3	△		32	●		
	H543	4	ㅋ		43	⊙		●
	H542	4	ㅋ		42	●		
	H532	3	△		32	●		
	H432	3	△		32	○		
73.75%	●		4	4		23	27	35
	○					8	8	
	□					2		
	△		46	47		2		
	ㄱ		7	44				
	●		32	33		35	35	35
	⊙		20	20		33	32	38

SUMMARY OF 3-4(5)

The situation regarding M_0 and M_1 is the same as that in Problem 3-4(4).

Reducing the advantage of M_1 from 17% to 10.5% gives a reduction in the efficiency of MM of 0.58% to 70.47%, the final order being:

1)	73.75%	COMBINE
2)	71.87%	QQ LL
3)	70.67%	QM LM
4)	70.47%	MM
5)	69.65%	REV MQ ML
6)	68.95%	CLA
7)	67.40%	BT QL LQ
8)	66.79%	MUD
9)	59.87%	JOU

PROBLEM 4-5(5)

In this problem there are four basic possibilities:

xxxx Hxxx
xxxxx Hxxxx

generating 21 possibilities in all.

The hidden hands contain the five lowest small cards:

6 5 4 3 2

Illustration:

J987

1. xxxx
2. xxxxx
3. Qxxx
4. Qxxxx

AK10

1. Qx
2. Q
3. xx
4. x

4-5 (5) **CLA**

⊗xxx	6543 6542 6532 6432 5432	6 Δ ● 6 Δ ● 6 Δ ● 6 Δ ● 5 ●	63 ● 62 Δ ● 62 Δ ● 62 Δ ●
⊗xxxx	65432	6 Δ ●	62 Δ ●
Hxx⊗	H654 H653 H652 H643 H642 H632 H543 H542 H532 H432	4 Δ 3 Δ 2 ● 3 Δ 2 ● 2 ● 3 Δ 2 ● 2 ● 2 ●	45 ● 35 ● 34 ● 34 ●
Hxx⊗x	H6543 H6542 H6532 H6432 H5432	4 Δ 4 Δ 3 Δ 3 Δ 3 Δ	43 ● 42 ● 32 ● 32 ● 32 ●
87.94 %	●	7 12	17 21 21
	○		
	□		
	Δ	14 9	4
	┐		
	⊕	21 21	21 21 21
	⊙	16 17	20 21 21

4-5 (5) **MUD**

⊗xxx	6543 6542 6532 6432 5432	6 ● 6 ● 6 ● 6 ● 5 Δ ●	 52 ●
x⊗xxx	65432	5 Δ ●	56 ●
Hxx⊗	H654 H653 H652 H643 H642 H632 H543 H542 H532 H432	4 Δ 3 Δ 2 ● 3 Δ 2 ● 2 ● 3 Δ 2 ● 2 ● 2 ●	45 ● 35 ● 34 ● 34 ●
Hxx⊗x	H6543 H6542 H6532 H6432 H5432	4 Δ 4 Δ 3 Δ 3 Δ 3 Δ	43 ● 42 ● 32 ● 32 ● 32 ●
88.36 %	●	10 12	21 21 21
	○		
	□		
	Δ	11 9	
	┐		
	⊕	21 21	21 21 21
	⊙	16 17	21 21 21

4-5(5) REV

x⊗xx	6543	5 △ •	56 •
	6542	5 △ •	56 •
	6532	5 △ •	56 •
	6432	4 ┐ •	46 •
	5432	4 ┐ •	45 ⊡ •
x⊗xxx	65432	5 △ •	52 •
Hxx⊗	H654	4 ┐ △	45 □ •
	H653	3 △	35 •
	H652	2 •	
	H643	3 △	34 •
	H642	2 •	
	H632	2 •	
	H543	3 △	34 •
	H542	2 •	
	H532	2 •	
	H432	2 •	
Hxx⊗x	H6543	4 ┐ △	43 •
	H6542	4 ┐ △	42 •
	H6532	3 △	32 •
	H6432	3 △	32 •
	H5432	3 △	32 •
84.60 %	•	6 12	13 21 21
	○		
	□		2
	△	10 9	
	┐	5	
	⊕	20 21	21 21 21
	⊙	11 17	20 21 21

4-5(5) BT QL

⊗xxx	6543	6 •	
	6542	6 •	
	6532	6 •	
	6432	6 •	
	5432	5 ┐ •	52 □ •
x⊗xxx	65432	5 ┐ •	56 •
Hx⊗x	H654	5 ┐ •	54 •
	H653	5 ┐ •	53 •
	H652	5 ┐ •	52 ⊡ •
	H643	4 •	
	H642	4 •	
	H632	3 △ •	32 •
	H543	4 •	
	H542	4 •	
	H532	3 △ •	32 •
	H432	3 △ •	32 •
Hxxx⊗	H6543	3 △ •	34 •
	H6542	2 •	
	H6532	2 •	
	H6432	2 •	
	H5432	2 •	
94.76 %	•	12 21	19 21 21
	○		
	□		2
	△	4	
	┐	5	
	⊕	20 21	21 21 21
	⊙	18 21	20 21 21

4-5(5) JOB

xx⊗x	6543	4 □	43 □ •
	6542	4 □	42 □ •
	6532	3 ┐ ⊡	32 ⊡ •
	6432	3 ┐ ⊡	32 ⊡ •
	5432	3 ┐ ⊡	32 ⊡ •
xxxx⊗	65432	2 □ •	26 •
Hx⊗x	H654	5 •	
	H653	5 •	
	H652	5 •	
	H643	4 ⊡	43 ⊡ •
	H642	4 ⊡	42 ⊡ •
	H632	3 ┐ □	32 □ •
	H543	4 ⊡	43 ⊡ •
	H542	4 ⊡	42 ⊡ •
	H532	3 ┐ □	32 □ •
	H432	3 ┐ □	32 □ •
Hxxx⊗	H6543	3 ┐ •	34 •
	H6542	2 ⊡ •	24 •
	H6532	2 ⊡ •	23 •
	H6432	2 ⊡ •	23 •
	H5432	2 ⊡ •	23 •
78.68 %	•	3 9	9 9 21
	○		
	□	11 12	12 12
	△		
	┐	7	
	⊕	20 21	21 21 21
	⊙	14 16	16 16 21

4-5(5) QQL

⊗xxx	6543	6 △ •	65 •
	6542	6 △ •	65 •
	6532	6 △ •	65 •
	6432	6 △ •	64 •
	5432	5 •	
⊗xxxx	65432	6 △ •	62 •
Hxx⊗	H654	4 •	
	H653	3 △	36 •
	H652	2 △	26 •
	H643	3 △	36 •
	H642	2 △	26 •
	H632	2 △	26 •
	H543	3 △	35 •
	H542	2 △	25 •
	H532	2 △	25 •
	H432	2 △	24 △ •
Hxxx⊗	H6543	3 △	34 •
	H6542	2 △	24 △ •
	H6532	2 △	23 •
	H6432	2 △	23 •
	H5432	2 △	23 •
85.40 %	•	2 7	19 21 21
	○		
	□		
	△	19 11	2
	┐		
	⊕	21 21	21 21 21
	⊙	15 16	20 21 21

4-5 (5) LLQ

⊗xxx	6543	6 □	65 ●
	6542	6 □	65 ●
	6532	6 □	65 ●
	6432	6 □	64 □ ●
	5432	5 □	54 ●
xxxx⊗	65432	2 □ ●	26 ●
H⊗xx	H654	6 ⊡	64 ⊡ ●
	H653	6 ⊡	63 ●
	H652	6 ⊡	62 ●
	H643	6 ⊡	63 ●
	H642	6 ⊡	62 ●
	H632	6 ⊡	62 ●
	H543	5 ⊡	53 ●
	H542	5 ⊡	52 ●
	H532	5 ⊡	52 ●
	H432	4 ●	
Hxxx⊗	H6543	3 ●	
	H6542	2 ⊡ ●	24 ●
	H6532	2 ⊡ ●	23 ●
	H6432	2 ⊡ ●	23 ●
	H5432	2 ⊡ ●	23 ●
85.40 %	●	2 7	19 21 21
	○		
	□	13 14	2
	△		
	⌝		
	⊕	21 21	21 21 21
	⊙	15 16	20 21 21

4-5 (5) MML

xxx⊗	6543	3 ⊙	36 ●
	6542	2 ⊙	26 ●
	6532	2 ⊙	26 ●
	6432	2 ⊙	26 ●
	5432	2 ⊙	25 ●
⊗xxxx	65432	6 ○ ●	62 ●
H⊗xx	H654	5 ⊙ ●	65 ●
	H653	6 ⊙ ●	65 ●
	H652	6 ⊙ ●	65 ●
	H643	6 ⊙ ●	64 ●
	H642	6 ⊙ ●	64 ●
	H632	6 ⊙ ●	63 ●
	H543	5 ●	
	H542	5 ●	
	H532	5 ●	
	H432	4 ●	
Hxxx⊗	H6543	3 ○	34 ●
	H6542	2 ○	24 ●
	H6532	2 ○	23 ●
	H6432	2 ○	23 ●
	H5432	2 ○	23 ●
90.90 %	●	4 11	21 21 21
	○	17 10	
	□		
	△		
	⌝		
	⊕	21 21	21 21 21
	⊙	17 18	21 21 21

4-5 (5) QM

x⊗xx	6543	5 ⊡	56 ●
	6542	5 ⊡	56 ●
	6532	5 ⊡	56 ●
	6432	4 □	46 ●
	5432	4 □	45 ●
⊗xxxx	65432	6 ●	
Hx⊗x	H654	5 □	54 ●
	H653	5 □	53 ●
	H652	5 □	52 ●
	H643	4 ⊡	43 ●
	H642	4 ⊡	42 ●
	H632	3 ◬ ●	32 ●
	H543	4 ⊡	43 ●
	H542	4 ⊡	42 ●
	H532	3 ◬ ●	32 ●
	H432	3 ◬ ●	32 ●
Hxxx⊗	H6543	3 △ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
85.82 %	●	5 9	21 21 21
	○		
	□	12 12	
	△	4	
	⌝		
	⊕	21 21	21 21 21
	⊙	15 16	21 21 21

4-5 (5) LQ

⊗xxx	6543	6 ●	
	6542	6 ●	
	6532	6 ●	
	6432	6 ●	
	5432	5 □ ●	52 ●
xxx⊗x	65432	3 ⌝ ●	32 ●
Hx⊗x	H654	5 ⊡ ●	56 ●
	H653	5 ⊡ ●	56 ●
	H652	5 ⊡ ●	56 ●
	H643	4 ●	
	H642	4 ●	
	H632	3 ⌝ ●	36 ●
	H543	4 ●	
	H542	4 ●	
	H532	3 ⌝ ●	35 ●
	H432	3 ⌝ ●	34 △ ●
Hxxx⊗	H6543	3 ⌝ ●	34 △ ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
94.76 %	●	12 21	19 21 21
	○		
	□	4	
	△		2
	⌝	5	
	⊕	20 21	21 21 21
	⊙	18 21	20 21 21

4-5(5) LM

x⊗xx	6543	5 ⊡	56 ●
	6542	5 ⊡	56 ●
	6532	5 ⊡	56 ●
	6432	4 ⊡	46 ●
	5432	4 ⊡	46 ●
xxx⊗x	65432	3 ⊡ ●	32 ●
H⊗xx	H654	6 ●	
	H653	6 ●	
	H652	6 ●	
	H643	6 ●	
	H642	6 ●	
	H632	6 ●	
	H543	5 □	53 ●
	H542	5 □	52 ●
	H532	5 □	52 ●
	H432	4 □	42 ●
Hxxx⊗	H6543	3 □ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
88.36%	●	10 12	21 21 21
	○		
	□	11 9	
	△		
	¬		
	⊕	21 21	21 21 21
	⊙	16 47	21 21 21

4-5(5) MQ

xx⊗x	6543	4 □	43 ●
	6542	4 □	42 ●
	6532	3 ¬ ⊡	32 ●
	6432	3 ¬ ⊡	32 ●
	5432	3 ¬ ⊡	32 ●
⊗xxxx	65432	6 ●	
Hx⊗x	H654	5 ●	
	H653	5 ●	
	H652	5 ●	
	H643	4 ⊡	46 ●
	H642	4 ⊡	46 ●
	H632	3 ¬ □	36 ●
	H543	4 ⊡	45 ●
	H542	4 ⊡	45 ●
	H532	3 ¬ □	35 ●
	H432	3 ¬ □	34 △ ●
Hxxx⊗	H6543	3 ¬ ●	34 △ ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
85.03%	●	8 9	19 21 21
	○		
	□	6 12	
	△		2
	¬	7	
	⊕	20 21	21 21 21
	⊙	15 16	20 21 21

4-5(5) ML

xx⊗x	6543	4 ⊡ ●	43 ●
	6542	4 ⊡ ●	42 ⊡ ●
	6532	3 ⊙ ●	32 ●
	6432	3 ⊙ ●	32 ●
	5432	3 ⊙ ●	32 ●
x⊗xxx	65432	5 ○ ●	56 ●
H⊗xx	H654	6 ●	
	H653	6 ●	
	H652	6 ●	
	H643	6 ●	
	H642	6 ●	
	H632	6 ●	
	H543	5 ⊙ ●	53 ●
	H542	5 ⊙ ●	52 ●
	H532	5 ⊙ ●	52 ●
	H432	4 □ ●	42 □ ●
Hxxx⊗	H6543	3 ○ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
95.13%	●	10 21	19 21 21
	○	8	
	□	3	2
	△		
	¬		
	⊕	21 21	21 21 21
	⊙	10 21	20 21 21

4-5(5) COMBINE

⊗xxx	6543	6 △ ●	63 ●
	6542	6 △ ●	62 ●
	6532	6 △ ●	62 ●
	6432	6 △ ●	62 ●
	5432	5 □ ●	52 □ ●
⊗xxxx	65432	6 △ ●	65 ●
Hx⊗x	H654	5 ⊡ ●	54 ●
	H653	5 ⊡ ●	53 ●
	H652	5 ⊡ ●	52 ⊡ ●
	H643	4 ●	
	H642	4 ●	
	H632	3 △ ●	32 ●
	H543	4 ●	
	H542	4 ●	
	H532	3 △ ●	32 ●
	H432	3 △ ●	32 ●
Hxxx⊗	H6543	3 △ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
95.13%	●	8 21	19 21 21
	○		
	□	4	2
	△	9	
	¬		
	⊕	21 21	21 21 21
	⊙	18 21	20 21 21

SUMMARY OF 4-5 (5)

Signals M_0 and M_1 only appear in the system MM ; M_0 having no negative influence while the advantage of M_1 relates to two successes of the type ⊙ in columns I_0 I_1.

Reducing this from 17 % to 10.5 % gives us an efficiency of 88.96 % for MM, the final order being:

1)	95.13 %	COMBINE ML
2)	94.76 %	BT QL LQ
3)	88.96 %	MM
4)	88.36 %	MUD LM
5)	87.94 %	CLA
6)	85.82 %	QM
7)	85.40 %	QQ LL
8)	85.03 %	MQ
9)	84.60 %	REV
10)	78.68 %	JOU

PROBLEM 5-6(6)

In this problem there are four basic possibilities:

xxxxx Hxxxx
xxxxxx Hxxxxx

generating 28 possibilities in all.

The hidden hands contain the six lowest small cards:

7 6 5 4 3 2

An illustration:

Q108
1. xxxxx
2. xxxxxx
3. Kxxxx
4. Kxxxxx
AJ9
1. Kx
2. K
3. xx
4. x

5-6 (6) CLA

⊗xxxx	76543	7 △ ●	73 ●
	76542	7 △ ●	72 △ ●
	76532	7 △ ●	72 △ ●
	76432	7 △ ●	72 △ ●
	75432	7 △ ●	72 △ ●
	65432	6 ●	
⊗xxxxx	765432	7 △ ●	72 △ ●
Hxx⊗x	H7654	5 △	54 ●
	H7653	5 △	53 △ ●
	H7652	5 △	52 △ ●
	H7643	4 △	43 ●
	H7642	4 △	42 △ ●
	H7632	3 ●	
	H7543	4 △	43 ●
	H7542	4 △	42 △ ●
	H7532	3 ●	
	H7432	3 ●	
	H6543	4 △	43 ●
	H6542	4 △	42 △ ●
	H6532	3 ●	
	H6432	3 ●	
	H5432	3 ●	
Hxx⊗xx	H76543	5 △	53 △ ●
	H76542	5 △	52 △ ●
	H76532	5 △	52 △ ●
	H76432	4 △	42 △ ●
	H75432	4 △	42 △ ●
	H65432	4 △	42 △ ●
83.17 %	●	7 13	12 17 28
	○		
	□		
	△	21 15	16 11
	┐		
	⊕	28 28	28 28 28
	◎	21 22	22 23 28

5-6 (6) MUD

x⊗xxx	76543	6 ●	
	76542	6 ●	
	76532	6 ●	
	76432	6 ●	
	75432	5 ┐ ●	57 ●
	65432	5 ┐ ●	56 ●
⊗xxxxx	765432	7 ●	
Hxx⊗x	H7654	5 ┐ △	54 ●
	H7653	5 ┐ △	53 △ ●
	H7652	5 ┐ △	52 △ ●
	H7643	4 △	43 ●
	H7642	4 △	42 △ ●
	H7632	3 ●	
	H7543	4 △	43 ●
	H7542	4 △	42 △ ●
	H7532	3 ●	
	H7432	3 ●	
	H6543	4 △	43 ●
	H6542	4 △	42 △ ●
	H6532	3 ●	
	H6432	3 ●	
	H5432	3 ●	
Hxx⊗xx	H76543	5 ┐ △	53 △ ●
	H76542	5 ┐ △	52 △ ●
	H76532	5 ┐ △	52 △ ●
	H76432	4 △	42 △ ●
	H75432	4 △	42 △ ●
	H65432	4 △	42 △ ●
81.83 %	●	11 13	17 17 23
	○		
	□		
	△	9 15	11 11
	┐	8	
	⊕	26 28	28 28 28
	◎	20 22	23 23 28

5-6 (6) REV

x⊗xxx	76543	6 △ ●	63 ●
	76542	6 △ ●	62 ●
	76532	6 △ ●	62 ●
	76432	6 △ ●	62 ●
	75432	5 ㄱ ●	52 ㅋ ●
	65432	5 ㄱ ●	52 ㅋ ●
x⊗xxxx	765432	6 △ ●	67 ●
Hxx⊗x	H7654	5 ㅋ △	54 ●
	H7653	5 ㅋ △	53 △ ●
	H7652	5 ㅋ △	52 ㄱ △ ●
	H7643	4 △	43 ●
	H7642	4 △	42 △ ●
	H7632	3 ●	
	H7543	4 △	43 ●
	H7542	4 △	42 △ ●
	H7532	3 ●	
	H7432	3 ●	
	H6543	4 △	43 ●
	H6542	4 △	42 △ ●
	H6532	3 ●	
	H6432	3 ●	
	H5432	3 ●	
Hxx⊗xx	H76543	5 ㅋ △	53 △ ●
	H76542	5 ㅋ △	52 ㅋ △ ●
	H76532	5 ㅋ △	52 ㅋ △ ●
	H76432	4 △	42 △ ●
	H75432	4 △	42 △ ●
	H65432	4 △	42 △ ●
80.00 %	●	6 13	15 17 28
	○		
	□		
	△	11 15	8 11
	ㄱ	8	5
	⊕	26 28	27 28 28
	⊙	19 22	21 23 28

5-6 (6) BT

x⊗xxx	76543	6 ⊙	67 ●
	76542	6 ⊙	67 ●
	76532	6 ⊙	67 ●
	76432	6 ⊙	67 ●
	75432	5 ○	57 ●
	65432	5 ○	56 ●
⊗xxxxx	765432	7 ●	
Hxxx⊗	H7654	4 ●	
	H7653	3 ●	
	H7652	2 ●	
	H7643	3 ●	
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 ●	
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 ●	
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hx⊗xxx	H76543	6 ○	63 ●
	H76542	6 ○	62 ●
	H76532	6 ○	62 ●
	H76432	6 ○	62 ●
	H75432	5 ⊙	52 ●
	H65432	5 ⊙	52 ●
92.38 %	●	16 16	28 28 28
	○	12 12	
	□		
	△		
	ㄱ		
	○	28 28	28 28 28
	⊙	24 24	28 28 28

5-6 (6) JOU

xxxx⊗	76543	3 □	37 ●
	76542	2 □	27 ●
	76532	2 □	27 ●
	76432	2 □	27 ●
	75432	2 □	27 ●
	65432	2 □	26 ●
xx⊗xxx	765432	5 □ ●	54 ●
Hxxx⊗	H7654	4 ●	
	H7653	3 ⊡	35 ●
	H7652	2 ⊡	25 ●
	H7643	3 ⊡	34 ●
	H7642	2 ⊡	24 ●
	H7632	2 ⊡	23 ●
	H7543	3 ⊡	34 ●
	H7542	2 ⊡	24 ●
	H7532	2 ⊡	23 ●
	H7432	2 ⊡	23 ●
	H6543	3 ⊡	34 ●
	H6542	2 ⊡	24 ●
	H6532	2 ⊡	23 ●
	H6432	2 ⊡	23 ●
	H5432	2 ⊡	23 ●
Hx⊗xxx	H76543	6 ●	
	H76542	6 ●	
	H76532	6 ●	
	H76432	6 ●	
	H75432	5 ⊡ ●	52 ●
	H65432	5 ⊡ ●	52 ●
87.46 %	●	5 8	28 28 28
	○		
	□	23 20	
	△		
	ㄱ		
	⊕	28 28	28 28 28
	⊙	24 22	28 28 28

5-6 (6) QQL

⊗xxxx	76543	7 △ ●	73 ●
	76542	7 △ ●	72 ●
	76532	7 △ ●	72 ●
	76432	7 △ ●	72 ●
	75432	7 △ ●	72 ●
	65432	6 ●	
⊗xxxxx	765432	7 △ ●	76 ●
Hxxx⊗	H7654	4 ●	
	H7653	3 △	35 ●
	H7652	2 △	25 ●
	H7643	3 △	34 ●
	H7642	2 △	24 ●
	H7632	2 △	23 ●
	H7543	3 △	34 ●
	H7542	2 △	24 ●
	H7532	2 △	23 ●
	H7432	2 △	23 ●
	H6543	3 △	34 ●
	H6542	2 △	24 ●
	H6532	2 △	23 ●
	H6432	2 △	23 ●
	H5432	2 △	23 ●
Hxxxx⊗	H76543	3 △	37 ●
	H76542	2 △	27 ●
	H76532	2 △	27 ●
	H76432	2 △	27 ●
	H75432	2 △	27 ●
	H65432	2 △	26 ●
87.46 %	●	2 8	28 28 28
	○		
	□		
	△	26 20	
	ㄱ		
	⊕	28 28	28 28 28
	⊙	21 22	28 28 28

5-6 (6) MML

⊗xxxx	76543	7 ◎	73 ●
	76542	7 ◎	72 ●
	76532	7 ◎	72 ●
	76432	7 ◎	72 ●
	75432	7 ◎	72 ●
	65432	6 ◎	62 ●
xxxxx⊗	765432	2 ○ ●	27 ●
Hxxx⊗	H7654	4 ●	
	H7653	3 ●	
	H7652	2 ◎ ●	25 ●
	H7643	3 ●	
	H7642	2 ◎ ●	24 ●
	H7632	2 ◎ ●	23 ●
	H7543	3 ●	
	H7542	2 ◎ ●	24 ●
	H7532	2 ◎ ●	23 ●
	H7432	2 ○ ●	23 ●
	H6543	3 ●	
	H6542	2 ◎ ●	24 ●
	H6532	2 ◎ ●	23 ●
	H6432	2 ◎ ●	23 ●
	H5432	2 ◎ ●	23 ●
H⊗xxx	H76543	7 ○	76 ●
	H76542	7 ○	76 ●
	H76532	7 ○	76 ●
	H76432	7 ○	76 ●
	H75432	7 ○	75 ●
	H65432	6 ○	65 ●
91.27%	●	5 16	28 28 28
	○	23 12	
	□		
	△		
	┐		
	○	28 28	28 28 28
	◎	23 24	28 28 28

5-6 (6) QL

x⊗xxx	76543	6 ●	
	76542	6 ●	
	76532	6 ●	
	76432	6 ●	
	75432	5 ●	
	65432	5 ●	
⊗xxxxx	765432	7 ●	
Hxxx⊗	H7654	4 △	45 ●
	H7653	3 △	35 ●
	H7652	2 ●	
	H7643	3 △	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 △	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 △	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hxxx⊗x	H76543	4 △	43 ●
	H76542	4 △	42 ●
	H76532	3 △	32 ●
	H76432	3 △	32 ●
	H75432	3 △	32 ●
	H65432	3 △	32 ●
90.48%	●	17 17	28 28 28
	○		
	□		
	△	11 11	
	┐		
	○	28 28	28 28 28
	◎	23 23	28 28 28

5-6 (G) QM

⊗xxxx	76543	7 ●	
	76542	7 ●	
	76532	7 ●	
	76432	7 ●	
	75432	7 ●	
	65432	6 △ ●	62 ●
x⊗::xxx	765432	6 △ ●	67 ●
Hxxx⊗	H7654	4 △	45 ●
	H7653	3 △	35 ●
	H7652	2 ●	
	H7643	3 △	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 △	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 △	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hxxx⊗x	H76543	4 △	43 ●
	H76542	4 △	42 ●
	H76532	3 △	32 ●
	H76432	3 △	32 ●
	H75432	3 △	32 ●
	H65432	3 △	32 ●
89.37%	●	15 17	28 22 28
	○		
	□		
	△	13 11	
	┐		
	○	28 28	28 28 28
	◎	22 23	28 28 28

5-6 (G) LQ

xxx⊗x	76543	4 ⊡	43 ●
	76542	4 ⊡	42 ●
	76532	3 ⊡	32 ●
	76432	3 ⊡	32 ●
	75432	3 ⊡	32 ●
	65432	3 ⊡	32 ●
⊗xxxxx	765432	7 ●	
Hxxx⊗	H7654	4 □	45 ●
	H7653	3 □	35 ●
	H7652	2 ●	
	H7643	3 □	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 □	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 □	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hx⊗xxx	H76543	6 ●	
	H76542	6 ●	
	H76532	6 ●	
	H76432	6 ●	
	H75432	5 ●	
	H65432	5 ●	
90.48 %	●	17 17	28 28 28
	○		
	□	11 11	
	△		
	┐		
	○	28 28	28 28 28
	◎	23 23	28 28 28

5-6 (6) LM

xxx⊗x	76543	4 ⊡	43 ●
	76542	4 ⊡	42 ●
	76532	3 ⊡	32 ●
	76432	3 ⊡	32 ●
	75432	3 ⊡	32 ●
	65432	3 ⊡	32 ●
x⊗xxxx	765432	6 ⊡ ●	67 ●
Hxxx⊗	H7654	4 □	45 ●
	H7653	3 □	35 ●
	H7652	2 ●	
	H7643	3 □	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 □	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 □	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
H⊗xxxx	H76543	7 ●	
	H76542	7 ●	
	H76532	7 ●	
	H76432	7 ●	
	H75432	7 ●	
	H65432	6 □ ●	62 ●
89.37 %	●	15 17	28 28 28
	○		
	□	13 11	
	△		
	˥		
	⊙	28 28	28 28 28
	◎	22 23	28 28 28

5-6 (6) MQ

⊗xxxx	76543	7 ●	
	76542	7 ●	
	76532	7 ●	
	76432	7 ●	
	75432	7 ●	
	65432	6 ○ ●	62 ●
xxxx⊗x	765432	3 ○ ●	32 ●
Hxxx⊗	H7654	4 ●	
	H7653	3 ⊙ ●	35 ●
	H7652	2 ●	
	H7643	3 ⊙ ●	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 ⊙ ●	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 ⊙ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hx⊗xxx	H76543	6 ⊙ ●	67 ●
	H76542	6 ⊙ ●	67 ●
	H76532	6 ⊙ ●	67 ●
	H76432	6 ⊙ ●	67 ●
	H75432	5 ●	
	H65432	5 ●	
97.78 %	●	18 28	28 28 28
	○	10	
	□		
	△		
	˥		
	⊙	28 28	28 28 23
	◎	26 28	28 28 28

5-6 (6) ML

x⊗xxx	76543	6 ⊙ ●	67 ●
	76542	6 ⊙ ●	67 ●
	76532	6 ⊙ ●	67 ●
	76432	6 ⊙ ●	67 ●
	75432	5 ●	
	65432	5 ●	
xxxx⊗x	765432	3 ○ ●	32 ●
Hxxx⊗	H7654	4 ●	
	H7653	3 ⊙ ●	35 ●
	H7652	2 ●	
	H7643	3 ⊙ ●	34 ●
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 ⊙ ●	34 ●
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 ⊙ ●	34 ●
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
H⊗xxxx	H76543	7 ●	
	H76542	7 ●	
	H76532	7 ●	
	H76432	7 ●	
	H75432	7 ●	
	H65432	6 ○ ●	62 ●
97.78 %	●	18 23	28 28 28
	○	10	
	□		
	△		
	˥		
	⊙	28 28	28 28 28
	◎	26 28	28 28 28

5-6 (6) COMBINE

⊗xxxx	76543	7 ●	
	76542	7 ●	
	76532	7 ●	
	76432	7 ●	
	75432	7 ●	
	65432	6 △ ●	65 ●
x⊗xxxx	765432	6 △ ●	62 ●
Hxxx⊗	H7654	4 △ ●	45 ●
	H7653	3 ●	
	H7652	2 ●	
	H7643	3 ●	
	H7642	2 ●	
	H7632	2 ●	
	H7543	3 ●	
	H7542	2 ●	
	H7532	2 ●	
	H7432	2 ●	
	H6543	3 ●	
	H6542	2 ●	
	H6532	2 ●	
	H6432	2 ●	
	H5432	2 ●	
Hxx⊗xx	H76543	5 ●	
	H76542	5 ●	
	H76532	5 ●	
	H76432	4 △ ●	42 ●
	H75432	4 △ ●	42 ●
	H65432	4 △ ●	42 ●
97.78 %	●	22 28	28 28 28
	○		
	□		
	△	6	
	˥		
	⊙	28 28	28 28 28
	◎	26 28	28 28 28

SUMMARY OF 5-6(6)

The situation regarding M_0 and M_1 is the same as that in Problem 4-5(5), except it applies to BT as well as MM.

Both their percentages are reduced by 1.46%, which makes the final order:

1)	97.78 %	COMBINE MQ ML
2)	90.92 %	BT
3)	90.48 %	QL LQ
4)	89.81 %	MM
5)	89.37 %	QM LM
6)	87.46 %	JOU QQ LL
7)	83.17 %	CLA
8)	81.83 %	MUD
9)	80.00 %	REV

N.B. COMBINE is slightly better than ML and MQ as there are more $\odot$.

SUMMARY

As a result of the analysis so far, we have a rough idea of the relative merits of various systems. If a system is at the top of the order in a problem, it is certainly a good one (e.g. C, MM); if it is near the bottom it is not so good (e.g. MUD, JOU). But to compare systems effectively we must calculate the average value of the efficiency of each system.

Frequency of problems

Assuming that the 6 problems analysed comprise 100% of all occurrences, their frequency is as follows:

$$\left.\begin{array}{l}\text{2-3 (4)} = 19.35\% \\ \text{2-3 (5)} = 18.68\%\end{array}\right\} 38.03\% \qquad \left.\begin{array}{l}\text{3-4 (4)} = 19.65\% \\ \text{3-4 (5)} = 24.23\%\end{array}\right\} 43.88\%$$

4-5 (5) = 13.98%

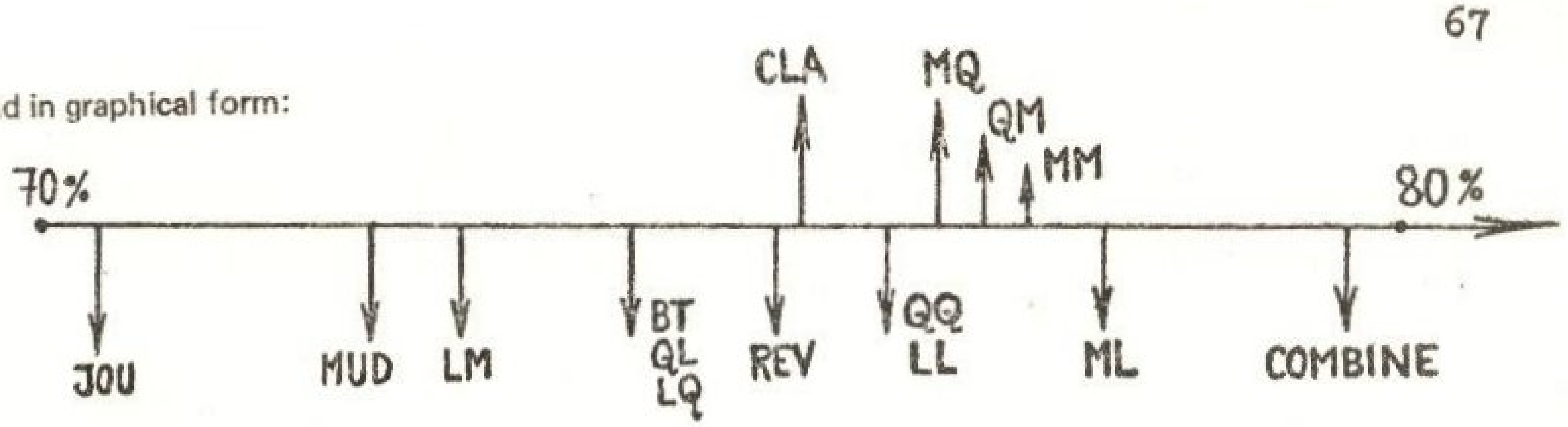
5-6 (6) = 4.11%

Final efficiency of systems

Using the above frequencies as weightings, we calculate the final efficiencies of systems as follows:

1)	79.56%	COMBINE	
2)	77.75%	ML	(Mixed-Length)
3)	77.20%	MM	(Mixed)
4)	76.94%	QM	(Quality-Mixed)
5)	76.56%	MQ	(Mixed-Quality)
6)	76.23%	QQ LL	(Quality) (Length)
7)	75.58%	CLA	(CLAssical)
8)	75.37%	REV	(REVers)
9)	74.40%	BT	(Blue Team)
10)	74.38%	QL LQ	(Quality-Length) (Length-Quality)
11)	73.09%	LM	(Length-Mixed)
12)	72.50%	MUD	
13)	70.37%	JOU	(JOUrnalist)

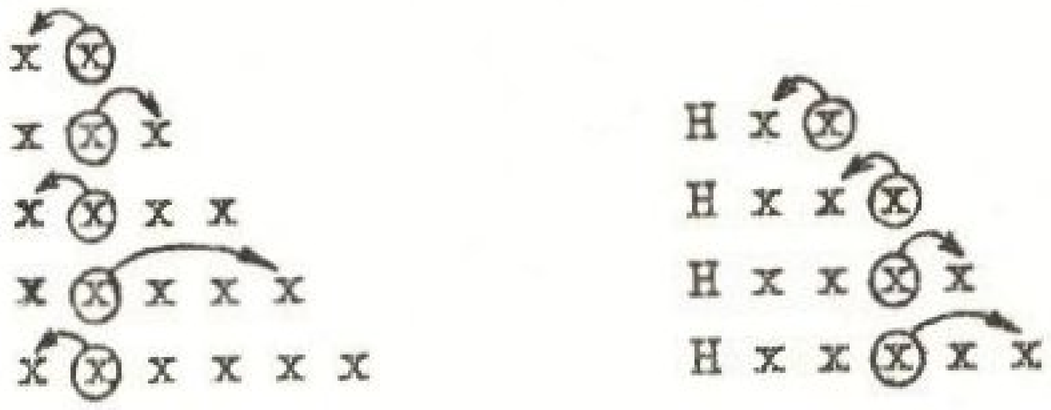
and in graphical form:



So we see that Combine is significantly better than the rest, and other new systems figure in the leading places (except LM QL LQ). It should be added that, apart from defining source S, systems QQ, LL and QL are not totally new, as:

QQ is based on giving quality signals
LL is based on giving length signals
QL is in principle the same as BT

When it comes to traditional and well-known systems it is surprising not so much that they occupy the bottom half of the table but that the newer the discovery, the lower the efficiency !! The oldest one (Classical) is the best, and the newest (Journalist) is the worst. Also, the first classical system was probably QQ, which means that introducing fourth-highest leads was not an improvement, but a step backwards ! The theory of small-card systems presented here enables us to avoid this kind of apparent improvement; every new discovery should be subjected to statistical analysis. As an example, let us analyse the idea of using classical leads from honours and reverse leads from small cards, or:



This idea arose through a misunderstanding. In 1975 the author tried playing the small-card system QM, which can be described as follows:

from small cards : QL*
from honours : QL

This was often explained somewhat inaccurately to opponents, i.e.:

from an honour : normal (length)
from small cards : reverse (length)

which was inaccurate as it only explained S_o , omitting S_F . Having explained the origin of the system "normal–reverse", let us now calculate its efficiency. It is equivalent to:

In problem 2 – 3	LM
3 – 4	atypical
4 – 5	REV
5 – 6	REV

So we have to construct a table for problem 3 – 4; this will give us efficiencies of 66.96%; 61.84%. Using this and the figures obtained from the tables in preceding chapters we get an overall efficiency of 67.71% – the worst system of all ! It is doubtful whether we would have noticed this without the help of statistical analysis, even after years of playing it.

The genesis of Combine

In 1974 I began work on the theory of systems, which ended successfully with the discovery of the Mixed Signal, the formulation of a general theory and the classification of small-card systems. From January 1975 onwards I started testing the system QM (which I felt was the best of the classifiable ones) in practice, simultaneously commencing a statistical analysis, the aim of which was to discover the optimum system. This analysis covered many different systems, not all of which have been mentioned in this book. On completion of the analysis in June 1975, I called the best of the systems Combination Leads, which I changed to Combine Leads in 1978. Combine, then, is the result of research, so its first place is not surprising. It is worth noting that Combine would still take first place (followed by ML and MQ) even if the signal M_4 is given the value of 50%.

.... .END OF PART TWO...

PART THREE

The COMBINE

HONOUR COMBINE

Leads from sequences

General rule:-

from a two-card sequence – the highest
from a broken sequence – the middle
from an interior sequence – the lowest

from a full sequence – the middle or the highest

Schematically:-

$\underline{H}H$
$H\underline{H}h$
$Hh\underline{h}$

$\underline{H}\,\underline{H}H$

From a full sequence the lead is:

the middle honour – when you want to emphasise possession of the lowest honour.
the highest honour – when the lowest honour is deemed unimportant.

A similar possibility exists when leading from a broken sequence:

From a broken sequence the highest honour can be led if there is no need to signal possession of the lowest honour.

An example: having ♢KQJxxx or ♢KQ10xxx on lead against a contract of 4♡, you should lead the king (as though you had KQxxxx), as it is doubtful whether the lowest honour will play a significant part in the play.

Leads from a doubleton sequence:

From a doubleton sequence (AK, KQ, QJ, J10, 109) against a suit contract the lower honour is led.

This suggests the possession of a third honour, so partner will tend to return the suit, thus increasing the chances of obtaining a ruff.

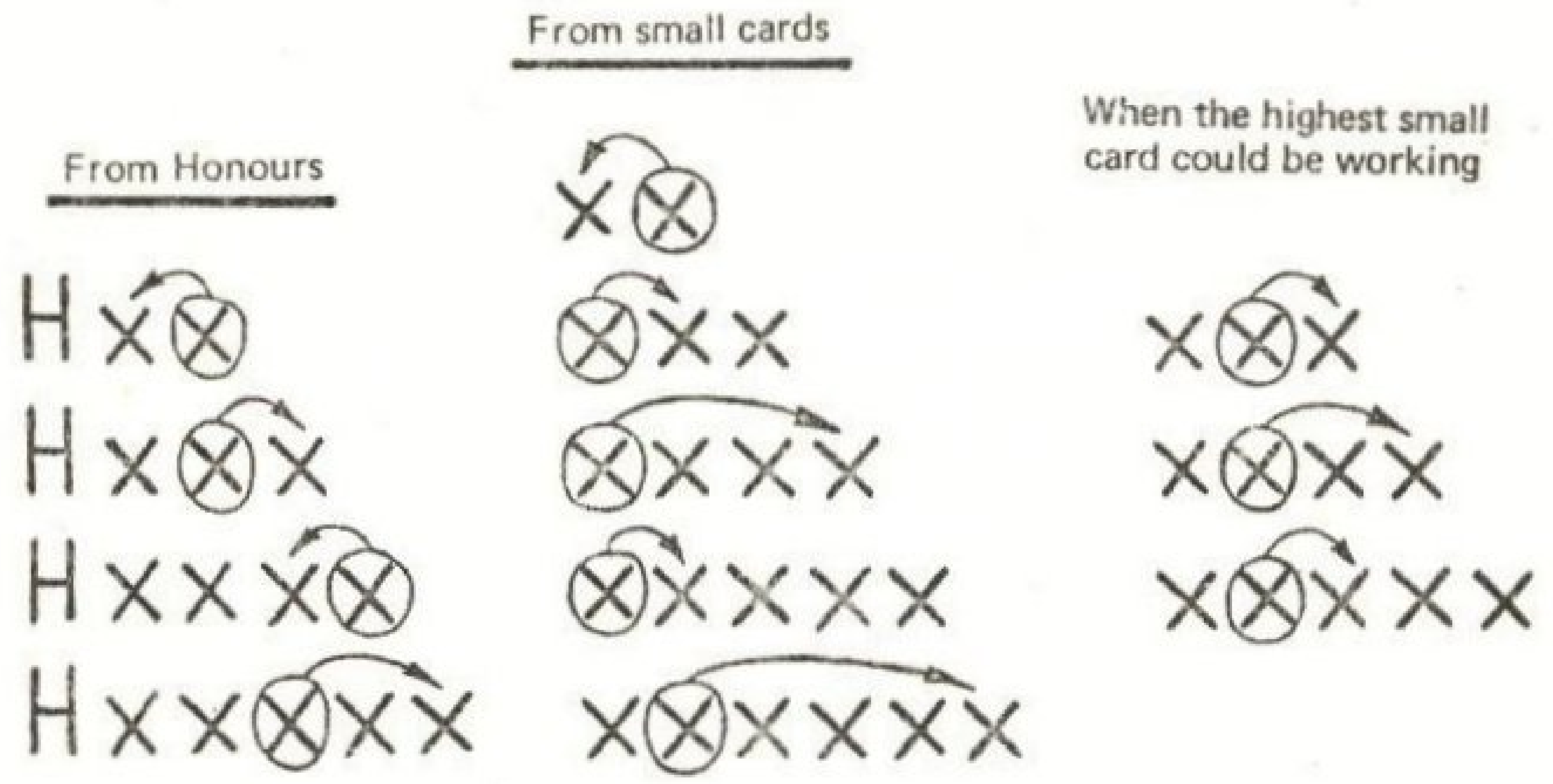
Leads from a doubleton honour

From the doubletons Ax Kx Qx Jx 10x 9x the honour is led, but against suit contracts the small card is led from 10x and 9x.

The reason for this is explained in the next chapter.

SMALL–CARD COMBINE

The scheme of playing small cards is this:



The plays in the third column would be used with these holdings: 10 7 3, 9 6 4 2, 10 8 7 6 3 etc. It should be borne in mind, however, that it is not only the rank of a small card which decides whether or not it is working, but also the expected distribution of the suit (based on the course of the auction). Also, as mentioned on page 12, the terms "honour" and "small card" should be used flexibly. For example: from a suit such as Jxxxx or Qxxxxx, against a suit contract, it may be right to lead as though the suit was xxxxx or xxxxxx. On the other hand, against no-trumps even the nine in a suit of 9xxxx may be treated as an honour.

THE MEANING OF LEADS

Let us now analyse the meaning of Combine leads from the point of view of the leader's partner.

Honour leads AKQJ

These show either a singleton or doubleton honour or these sequences:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
A =	AK					AKQ	AKJ
K =	KQ	AK	AKJ		AKQ	KQJ	KQ10
Q =	QJ	KQ	KQ10		KQJ	QJ10	QJ9
J =	J10	QJ	QJ9	AQJ	QJ10		

The above possibilities are arranged in columns, starting with the most probable:

1) From a two-card sequence
2) From a doubleton sequence against a suit contract
3) From a broken sequence
4) From an interior sequence
5) 6) From a full sequence
7) From a broken sequence (occasionally)

Leads of the 10 and 9

The 10 and 9 can be treated in two ways; either as honours or as small cards, depending on the situation. Their meanings, then, consist of possibilities from both Combines – Honour and Small-card:

	From at least three small cards	From an interior sequence	From a doubleton in no-trumps
10 =	10xx...	AJ10 KJ10	10x
9 =	9xx...	A109 K109 Q109	9x

Leads of small cards

Remember that the 10 and 9 are also treated as small cards, albeit high ones.

First small card (F), or the first card played:

low (↓) = agressive lead (xx or Hxx.......)
high (↑) = passive lead (xxx.........)

So the first small card is, as you can see, something in the nature of encouragement / discouragement in the suit led.

Second small card (S) has different meanings depending on the type of lead.

Second small card after a passive lead (xxx.......)
Is always lower than the first (order ↷)
Its rank transmits a reverse length signal (L*):
high (↑) = odd number of cards
low (↓) = even number of cards

Combine using normal length signals would be equally effective here.

Second small card after an agressive lead
Is always the lowest !
The only source of further information is comparison of the rank of the two cards (order). This order transmits a mixed signal (M):
upwards (↶) = even number of small cards
downwards (↷) = odd number of small cards

SIGNALS

It would be desirable for signals when following suit to be identical to leads, even if only to lessen the amount of memory work. As yet, however, this possibility has not been examined thoroughly enough by the author. Perhaps readers could experiment with it ? In the meantime it is recommended to use the following signals when playing Combine:

Mixed Signal (preferred)
Quality Signal
Reverse Length Signal

Reverse Smith Peters
Lavinthal

Use of signals

This is only defined in certain circumstances. In the remaining situations a common-sense rule applies: Give that signal which, in your opinion, partner needs.

Mixed Signal

Preferred and so most often used:
↶ or ↓ = even number of small cards
↷ or ↑ = odd number of small cards

If the quality of the suit is known, then the Mixed Signal becomes a length signal:
normal for a suit of good quality
reverse for a suit of bad quality

If the length of the suit is known, then the Mixed Signal becomes a quality signal:
normal for an odd number of cards
reverse for an even number of cards

The advantage of the Mixed Signal over length and quality signals has been demonstrated on page 17.

Quality Signal

This is only used when it is clear that partner needs information about quality:
↶ or ↓ = good quality (encouragement)
↷ or ↑ = bad quality (discouragement)

N.B. This method is popularly known as "reverse", but, as the author has shown on page 21, it is really "normal".

Reverse Length Signal

This is used only when it is clear that partner needs information about length:
↶ or ↓ = even number of cards
↷ or ↑ = odd number of cards

N.B. Reverse length signals are mistakenly thought of by many players as the same as the Reverse System, because of the similarity of their names. In fact, a reverse length signal (just like any other signal) should be given as clearly as possible, i.e. beginning with the extreme (lowest or highest) small card:

Reverse Smith Peters

If the situation in the first suit led by the defence is not yet clear, then in the first suit declarer plays, the opening leader plays:
↓ or ↶ = encouragement (continue)
↑ or ↷ = discouragement (switch)

Lavinthal

This is used whenever:
1) It is clearly necessary
2) No other signal makes sense

.............................END OF PART THREE.............................

* * *

AFTERWORD

In spite of the mass of theory it contains, this book has by no means covered all the new possibilities in the field of defence. For example, you could give information as to both sequences and number of cards simultaneously. Although the author has not fully developed this idea, one simple sequential-length system called "SEQUEL" is:

Leads from sequences depend on the number of cards in the suit:
odd : $\underline{H}H$ $H\underline{H}h$ $Hh\underline{h}$ (Combine)
even : $H\underline{H}$ $\underline{H}Hh$ $H\underline{h}h$ (Anti-combine !)

If you enjoy experimenting, try it.

.............................T H E   E N D.............................

The ADVANTAGES

OF BIDDING FREEDOM AND WEAK OPENING SYSTEMS IN DUPLICATE BRIDGE

(first published: December 1978)

Is the game of bridge just a matter of playing the cards well and choosing the best available bid within the confines of some standard system?

No! Certainly NOT.

Bridge is also a contest between bidding systems, depending on the invention of your own conventions, which in the long run will hopefully prove superior to those of your opponents:

AN INTEGRAL PART OF BRIDGE
IS COMPETITION
IN THE DEVELOPEMENT
OF BIDDING SYSTEMS

Inventing and testing bidding systems is not just one aspect of the game.

It is also an incredibly stimulating intellectual pastime! You can develop your own theories and immediately test them at the bridge table!

That is why players the world over invent their own conventions.

That is why nearly all serious bridge players begin their careers by inventing their own bidding systems.

That is the way the author of this booklet began some 20 years ago, and in spite of lack of major successes he has not become disillusioned with bridge – precisely because he was able to test his theories.

There is therefore no doubt that:

THE OPPORTUNITY FOR INVENTING
AND TESTING OF BIDDING SYSTEMS
AND CONVENTIONS IS ONE OF THE
MAIN ATTRACTIONS OF BRIDGE

Imagine the following situation: – a youngish player has invented a bidding system and has decided to try it out.

Where is he to do this?

Not in a teams-of-4 match, because if his system lets him down, his teammates will be somewhat annoyed – and rightly so.

That leaves a pairs tournament, where his only responsibility is to his partner, and a poor performance will not eliminate him from further events.

From this it follows that:

THERE SHOULD BE NO RESTRICTIONS
ON BIDDING SYSTEMS AND CONVENTIONS
AT TOURNAMENTS AND CONGRESSES

At this point, some people might say that all these "inventions" are practically useless, and that one should stick to the old, tried and trusted bidding methods.

Suppose, then, that all new conventions have been banned and everyone uses identical (or nearly identical) system.

What happens?

1. Boredom sets in,
2. Smaller attendances at tournaments,
3. Bidding theory stagnates!

Let us not forget that new ideas are born from experiments!

Don't disparage the players who use systems which at first sight appear totally senseless. Without them we would still be bidding the way we did 50 years ago!

So we see that:

FREEDOM TO USE ANY BIDDING SYSTEM
IS VITAL TO THE DEVELOPEMENT
OF BIDDING THEORY

The possibility of developing and testing bidding systems is a characteristic of the game of bridge, distinguishing it from all other intellectual sports!

Why, then, should we diminish the game by imposing on its players laws which make bidding less interesting?

THE BIDDING FREEDOM
IS ADVANTAGEOUS
FOR BRIDGE

THE ADVANTAGES OF WEAK OPENING SYSTEMS

As recently as 10 years ago, almost everyone used the same basic bidding methods; only a handful of players used Weak Opening Systems.

There were many traditional systems, of course, but in relative terms these differed only slightly, and this is still true today.

However, Weak Opening Systems are something completely different: they destroy the foundations on which the traditional systems are built and erect new ones in their place.

Are Weak Opening Systems good or bad for bridge?

Let us examine the matter.

The main characteristic of Weak Opening Systems are:

1) A high frequency of opening bids (80% of hands)
2) Opening the bidding with weak hands
3) Unusual methods of describing distribution.

These characteristics make bridge a much more interesting game, as:

YOU DON'T GET BORED!

You enter the bidding on nearly every hand – even with a yarborough.

OPPONENTS' BIDDING BECOMES MORE DIFFICULT!

And this merely because you have opened the bidding (it is well known that defensive bidding is difficult, even for experts).

YOU ARE INTELLECTUALLY STIMULATED!

The unusual and original nature of Weak Opening Systems makes them an interesting intellectual pastime, providing you with freshness and novelty.

Hence, it is obvious that:

WEAK OPENING SYSTEMS
MAKE BRIDGE
A MORE INTERESTING GAME

The attraction of Weak Opening Systems has resulted in a constant growth in their popularity, to such an extent that in Poland at present they are seriously challenging orthodox systems.

Weak Opening Systems are rarely mentioned in the bridge press, no experts use them at the highest levels of the game, and yet in Warsaw alone there are over 200 players who use them!

And the majority of those only became interested in bridge after they discovered Weak Opening Systems!

From this we can draw the conclusion that:

WEAK OPENING SYSTEMS
HELP TO RECRUIT NEW PLAYERS
FOR DUPLICATE BRIDGE

Should you doubt the added interest of Weak Opening Systems, remember the last time you found yourself in this situation:

It's one of those days when you seem to pick up the same 5 or 6 count on every hand; you continually pass with ever-increasing despondency. Not only you are bored to tears, but worse, you have no control over your results, and are reduced to hoping opponents have a bidding accident or pull the wrong card out.

It's different with Weak Opening Systems!

More than ever, the result of a tournament becomes independent of how good your hands are.

This is because:

1) You open the bidding very frequently (always if you get a chance)
2) The meaning of opening bids is often unusual, though easy to comprehend.

In effect, the opponents are reduced to bidding defensively, difficult even for experts.

Thus, Weak Opening Systems deprive seasoned pleyers of the advantage they would normally enjoy due to their experience.

They have to play as well as they can, with great care. They cannot sit back and relax, counting on beating their inexperienced opponents without too much effort.

So:

DUPLICATE BRIDGE
BECOMES EVEN MORE
OF A TEST OF SKILL

Weak Opening Systems are a new stage in the evolution of bridge theory – new axioms, new methods, a new style of bidding.

15 years ago they were germinating; now they are overrunning Poland, and germinating abroad; and in 15 years time they wil conquere the world!

Not because they are better than traditional systems, but above all because, thanks to them, bridge becomes a more interesting game:

You don't get bored when you hold poor cards!
You're always in the bidding!
You can compete effectively with top players!

WEAK OPENING SYSTEMS
ARE GOOD
FOR DUPLICATE BRIDGE

NO RESTRICTIONS!

In the past, bridge administrators have often tried to ban various new systems.

Who knows what Culbertson would have done, had the organisation of the game been the same as it is now?

Perhaps he would have been banned from opening 1♠ with a suit weaker than AKDxx?

The law-makers produce numerous arguments, such as:

– unusual systems come as a surprise to opponents
– it is difficult to agree an ad hoc defense to them
– it is harder to detect cheats

...and so on.

All these pales into insignificance beside the arguments for being able to use any bidding system.

So I repeat:

ANY RESTRICTION
ON BIDDING SYSTEMS
IS HARMFUL
TO DUPLICATE BRIDGE

Let us not deprive players of one of the main attractions of bridge, and the reason it is at the forefront of intellectual games – the chance to invent and test one's own conventions and systems.

POSTSCRIPT 1

If you had intended to introduce (or maintain) restrictions on bidding systems, please re-read the foregoing text and consider whether are you playing into the hands of people who are smugly satisfied with the status quo and are too lazy to use their brains.

Remember also that these restrictions produce a chain reaction – if you introduce them others will too, citing you as a precedent.

POSTSCRIPT 2

If in spite of this you decide to introduce a restriction on bidding systems in your tournament, do not be ashamed to publish this fact in your brochure.

Do not subject players who in many cases, have travelled a long way to play in your tournament to indignity of finding out about these restrictions only when they pull their cards out of the first board.

* * * * *